From isospin-related phenomena to stellar weak processes within beyond-mean-field approach

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Outline

- complex EXCITED VAMPIR beyond-mean-field model

- shape-coexistence and isospin-symmetry-breaking effects in the A=74 isovector triplet on:
  - Coulomb energy differences (CED)
  - mirror energy differences (MED)
  - triplet energy differences (TED)
  - superallowed Fermi $\beta$-decay of the $Z=N+2$ isotope $^{74}\text{Sr}$

- shape-coexistence effects on terrestrial and stellar weak interaction rates for
  - $^{74}\text{Sr}$
  - $^{68}\text{Se}$ rp-process waiting point
A~70 proton-rich nuclei manifest exotic structure and dynamics generated by the interplay of:

- shape coexistence and shape mixing
- competing $T=0$ and $T=1$ pairing correlations
- isospin-symmetry-breaking interactions

responsible for

drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods
- comprehensive understanding of structure phenomena and $\beta$-decay properties
- reliable predictions on stellar weak interaction rates

  based on

  self-consistent description of experimentally accessible properties
complex VAMPIR model family

- the model space is defined by a finite dimensional set of spherical single particle states

- the effective many-body Hamiltonian is represented as a sum of one- and two-body terms

- the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua

- the HFB transformations are essentially complex and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry (T=1 and T=0 neutron-proton pairing correlations already included at the mean field level)

- the broken symmetries (s=N, Z, I, p) are restored by projection before variation

* The models allow to use rather large model spaces and realistic effective interactions
Beyond-mean-field variational procedure

complex Vampir

\[ E^s[F^s_1] = \frac{\langle F^s_1| \hat{H} \Theta^s_{00} | F^s_1 \rangle}{\langle F^s_1| \Theta^s_{00} | F^s_1 \rangle} \]

\[ |\psi(F^s_1); sM\rangle = \frac{\Theta^s_{M0} |F^s_1\rangle}{\sqrt{\langle F^s_1| \Theta^s_{00} | F^s_1 \rangle}} \]

complex Excited Vampir

\[ |\psi(F^s_i); sM\rangle = \sum_{j=1}^{i} |\phi(F^s_j)\rangle \alpha^j_i \quad \text{for } i = 1, \ldots, n - 1 \]

\[ |\phi(F^s_i); sM\rangle = \Theta^s_{M0} |F^s_i\rangle \]

\[ |\psi(F^s_n); sM\rangle = \sum_{j=1}^{n-1} |\phi(F^s_j)\rangle \alpha^j_n + |\phi(F^s_n)\rangle \alpha^n_n \]

\[ (H - E^{(n)} N) f^n = 0 \]

\[ (f^{(n)})^+ N f^{(n)} = 1 \]

\[ |\Psi_{\alpha}^{(n)}; sM\rangle = \sum_{i=1}^{n} |\psi_i; sM\rangle f^{(n)}_{i\alpha}, \quad \alpha = 1, \ldots, n \]
$A \sim 70$ mass region

$^{40}\text{Ca} - \text{core}$

model space for protons and neutrons:

$1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 1d_{5/2} \ 0g_{9/2}$

(charge-symmetric basis + core Coulomb contributions to proton single particle energies)

renormalized $G$-matrix (Bonn A/CD potential)

- pairing properties enhanced by short range Gaussians for:
  
  $T = 1$ : $pp \ (-35 \text{ MeV})$, $np \ (-20 \text{ MeV})$, $nn \ (-35 \text{ MeV})$
  
  $T = 0$, $S = 0$ and $S = 1 \ (-35 \text{ MeV})$

- onset of deformation influenced by monopole shifts:
  
  $<0g_{9/2} \ 0f; T=0 \ |G| \ 0g_{9/2} \ 0f; T=0> \ (0f_{5/2}, 0f_{7/2})$
  
  $<1d_{5/2} \ 1p; T=0 \ |G| \ 1d_{5/2} \ 1p; T=0> \ (1p_{1/2}, 1p_{3/2})$

- Coulomb interaction between valence protons added
Isospin-symmetry breaking and shape coexistence in the $A=74$ isovector triplet

$^{38}\text{Sr}_{36} - ^{37}\text{Rb}_{37} - ^{36}\text{Kr}_{38}$


**Coulomb Energy Differences**

$CED_{J,T=1} = E^*_{J,T=1,Tz=0} - E^*_{J,T=1,Tz=+1}$

**spectroscopic quadrupole moments**

<table>
<thead>
<tr>
<th>$I(\hbar)$</th>
<th>$^{74}\text{Kr}$ Exp</th>
<th>$^{74}\text{Rb}$</th>
<th>$^{74}\text{Sr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_1$</td>
<td>-54</td>
<td>-57</td>
<td>-50</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>49</td>
<td>53</td>
<td>48</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>-74</td>
<td>-77</td>
<td>-70</td>
</tr>
<tr>
<td>$4^+_2$</td>
<td>68</td>
<td>72</td>
<td>67</td>
</tr>
<tr>
<td>$6^+_1$</td>
<td>-85</td>
<td>-86</td>
<td>-81</td>
</tr>
<tr>
<td>$6^+_2$</td>
<td>78</td>
<td>81</td>
<td>80</td>
</tr>
</tbody>
</table>
$A = 74$: wave functions reveal shape mixing

- significant oblate-prolate mixing decreasing with increasing spin

<table>
<thead>
<tr>
<th>$^74\text{Kr}$</th>
<th>$I(\hbar)$</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>82(1)(1) %</td>
<td>14(1)(1) %</td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>92(1)(1) %</td>
<td>6 %</td>
<td></td>
</tr>
<tr>
<td>$4^+$</td>
<td>95(1)(1) %</td>
<td>3 %</td>
<td></td>
</tr>
<tr>
<td>$6^+$</td>
<td>97(1) %</td>
<td>1(1) %</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$^74\text{Rb}$</th>
<th>$I(\hbar)$</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>85(1) %</td>
<td>12(1) %</td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>94(1) %</td>
<td>4 %</td>
<td></td>
</tr>
<tr>
<td>$4^+$</td>
<td>96(1) %</td>
<td>2 %</td>
<td></td>
</tr>
<tr>
<td>$6^+$</td>
<td>97(1) %</td>
<td>1 %</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$^74\text{Sr}$</th>
<th>$I(\hbar)$</th>
<th>Prolate content</th>
<th>Oblate content</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>77(2) %</td>
<td>19(1) %</td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>87(1) %</td>
<td>11 %</td>
<td></td>
</tr>
<tr>
<td>$4^+$</td>
<td>90(1) %</td>
<td>8 %</td>
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</tr>
<tr>
<td>$6^+$</td>
<td>92(1) %</td>
<td>5(1) %</td>
<td></td>
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</tbody>
</table>

- maximum oblate-prolate mixing in $^74\text{Sr}$
Isospin-symmetry-breaking and shape-mixing effects on

Mirror Energy Differences & Triplet Energy Differences

Charge-symmetry breaking:
\[ V^{(1)}_{\text{INC}} = V_{pp} - V_{nn} \]
\[ (V_{nn} 1\%\text{ stronger than } V_{pp}) \]

\[ \text{MED}_{J,T=1} = E^*_{J,T=1,T_z=-1} - E^*_{J,T=1,T_z=+1} \]

Charge-independence breaking:
\[ V^{(2)}_{\text{INC}} = V_{pp} + V_{nn} - 2 V_{pn} \]
\[ (V_{pn} 2.5\%\text{ stronger than the average of } V_{pp} \text{ and } V_{nn}) \]

\[ \text{TED}_{J,T=1} = E^*_{J,T=1,T_z=-1} + E^*_{J,T=1,T_z=+1} - 2E^*_{J,T=1,T_z=0} \]

A=74: complex Excited Vampir predictions

experimental results (J. Henderson, Phys. Rev. C 90, 051303(R) (2014))
Self-consistent terrestrial and stellar weak interaction rates

Fermi transition probabilities

\[ B_{if}(F) = \frac{1}{2J_i + 1} \frac{g_V^2}{4\pi} |M_F|^2 \]

\[ M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i) \]

\[ = \delta J_i J_f \sum_{ab} M_F(ab)(\xi_f J_f || [c_a^{\dagger} \tilde{c}_b]_0 || \xi_i J_i) \]

\[ M_F(ab) = (a || \hat{1} || b) \]

Gamow-Teller transition probabilities

\[ B_{if}(GT) = \frac{1}{2J_i + 1} \frac{g_A^2}{4\pi} |M_{GT}|^2 \]

\[ M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i) \]

\[ = \sum_{ab} M_{GT}(ab)(\xi_f J_f || [c_a^{\dagger} \tilde{c}_b]_1 || \xi_i J_i) \]

\[ M_{GT}(ab) = 1/\sqrt{3}(a || \hat{\sigma} || b) \]

Superallowed Fermi β-decay for $^{74}$Sr: isospin-symmetry-breaking and shape-coexistence effects


\[ f'(1 + \delta_R)(1 - \delta_c) = \frac{K}{2G_V^2(1 + \Delta_{RI})} \]

\( \delta_c \) – isospin-symmetry-breaking correction

\[ Q_{EC} = 11.090 \text{ MeV} \]

\[ 1% \leq \delta_c \leq 3% \]

Nonanalog branches: $0^+_{II}, 0^+_{VI} \leq 0.8 \%$
Fermi $\beta$-decay of low-lying $2^+_{\text{yrast}}, 0^+_{\text{exc}}, 2^+_{\text{sec}}$ states with possible relevance for the rp-process path

$1\% \leq \delta_c \leq 3.6\%$

Nonanalog branches:

$2^+_{II} \leq 1.3\%$, $2^+_{IV} \leq 0.8\%$
Gamow-Teller $\beta$-decay and shape coexistence for $^{74}\text{Sr}$


Independent chains of variational calculations in parent and daughter nuclei

Large variety of deformations for daughter states revealed by spectroscopic quadrupole moments

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**Gamow-Teller states**

- **1$^+$ Gamow-Teller states**
  - $^{74}\text{Rb}$
  - Excitation energy (MeV)
  - $Q_{sp}$ (em$^2$)

- **3$^+$ Gamow-Teller states**
  - $^{74}\text{Rb}$
  - Excitation energy (MeV)
  - $Q_{sp}$ (em$^2$)
Gamow-Teller strength distributions for the decay of $0^+$ and $2^+$ states in $^{74}\text{Sr}$

Strength distributions reveal specific shape mixing for the parent states
Influence of shape mixing in parent and daughter states on strength distributions

Contributions from $p^{\nu(\pi)}_{1/2} p^{\pi(\nu)}_{3/2}$, $p^{\nu}_{3/2} p^{\pi}_{3/2}$, $f^{\nu}_{5/2} f^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2} f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2} g^{\pi}_{9/2}$ matrix elements (coherent / cancelling effect)
Terrestrial half-lives

\[
\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0<E_f<Q_{EC}} f(Z, E_f) [B_{if}(GT) + B_{if}(F)]
\]

\[
T_{1/2}^{GT} = 137 \text{ ms} \quad T_{1/2}^F = 48 \text{ ms}
\]

\[
T_{1/2}^{\text{EXVAM}} = 36 \text{ ms} \quad T_{1/2}^{\text{exp}} = 27(8) \text{ ms}
\]
Weak interaction rates in X-ray burst astrophysical environment

In the X-ray burst stellar environment at densities (~$10^6$ mol/cm$^3$) and temperatures (~$10^9$K) typical for the rp-process the contribution of thermally populated low-lying $0^+$ and $2^+$ states may be relevant.


\[
\chi^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \phi_{ij}^\alpha
\]

\[
G(Z, A, T) = \sum_i (2J_i + 1) \exp\left(-\frac{E_i}{kT}\right)
\]

\[
B_{ij} = B_{ij}(F') + B_{ij}(GT)
\]

\[
\phi_{ij}^{ec} = \int_{w_l}^{\infty} wp(Q_{ij} + w)^2 F(Z, w)S_e(w)(1 - S_\nu(Q_{ij} + w))dw
\]

\[
\phi_{ij}^{\beta^+} = \int_{1}^{Q_{ij}} wp(Q_{ij} - w)^2 F(-Z + 1, w)(1 - S_p(w))(1 - S_\nu(Q_{ij} - w))dw
\]
Stellar rates for $^{74}\text{Sr}$ : $\beta^+$ - decay

$E_{0_{\text{exc}}^+}^{\text{th}} = 0.564 \text{ MeV} \quad E_{2_{\text{yrast}}^+} = 0.471 \text{ MeV} \quad E_{2_{\text{sec}}^+}^{\text{th}} = 0.823 \text{ MeV}$
$\beta^+$ and electron capture rates for $^{74}\text{Sr}$
Weak interaction rates and shape coexistence for $^{68}$Se waiting point


Shape coexistence and strong mixing in parent and daughter states

$^{68}$Se:

- $E_{0^+}^{\text{gs}} = 0.0$ MeV [62/55(\%) – oblate (BonnA/BonnCD)]
- $E_{2^+}^{\text{yr}} = 0.854$ MeV [60/41(\%) – oblate (BonnA/BonnCD)]
- $Q^{s\gamma}_{2^+} = 3.5$ efm$^2$(A) / -7.1 efm$^2$(CD)
- $B(E2;2^+\rightarrow 0^+) = 521/503$ e$^2$fm$^4$ (BonnA/BonnCD) Exp.: 430(60) e$^2$fm$^4$
Excitation energy (MeV)

$^{68}\text{Se}$ EXVAM-Bonn A

$2^+_\nu \rightarrow 1^+$

$10^0 \cdot \text{B}(\Delta^2)$ (\text{g.e.})

Excitation energy (MeV)

$^{68}\text{Se}$ EXVAM-Bonn CD

$2^+_\nu \rightarrow 1^+$

$10^0 \cdot \text{B}(\Delta^2)$ (\text{g.e.})

Contributions:

- $p^{\nu(\pi)}\frac{1}{2}p^{\nu(\pi)}\frac{3}{2}$, $p^{\nu(\pi)}\frac{3}{2}p^{\pi}\frac{3}{2}$, $f^{\nu(\pi)}\frac{5}{2}f^{\pi}\frac{5}{2}$, $f^{\nu(\pi)}\frac{5}{2}f^{\pi(\nu)}\frac{7}{2}$, $g^{\nu(\pi)}g^{\pi(\nu)}\frac{9}{2}$ matrix elements (decay to $1^+$ states)

- $p^{\nu(\pi)}\frac{3}{2}p^{\nu(\pi)}\frac{1}{2}$, $p^{\nu(\pi)}\frac{3}{2}p^{\pi}\frac{3}{2}$, $f^{\nu(\pi)}\frac{5}{2}f^{\pi}\frac{7}{2}$ matrix elements (decay to $3^+$ states)

$^{68}\text{Se}$ Excitation energy (MeV)

$\sum \text{B}(\Delta^2) (\text{g.e.})$

$\nu_{\text{EC}} \downarrow$

$^{68}\text{Se}$ EXVAM-BonnA - full line

$^{68}\text{Se}$ EXVAM-BonnCD - dashed line

$T_{1/2}^{\text{EXVAM}} = 48.8 / 33.5$ ms (BonnA/BonnCD) $T_{1/2}^{\text{exp}} = 35.5(7)$ ms
Stellar rates for $^{68}$Se: $\beta^+$ and continuum electron capture

Significant continuum electron capture contribution
Summary

complex EXCITED VAMPIR beyond-mean-field model self-consistently describes shape-coexistence effects on

- isospin-related phenomena in the $A=74$ isovector triplet:
  CED, MED, TED, superallowed Fermi $\beta$-decay

- terrestrial and stellar weak interaction rates for $A\sim70$ proton-rich nuclei