Chiral basis for particle-rotor model for odd-odd triaxial nuclei

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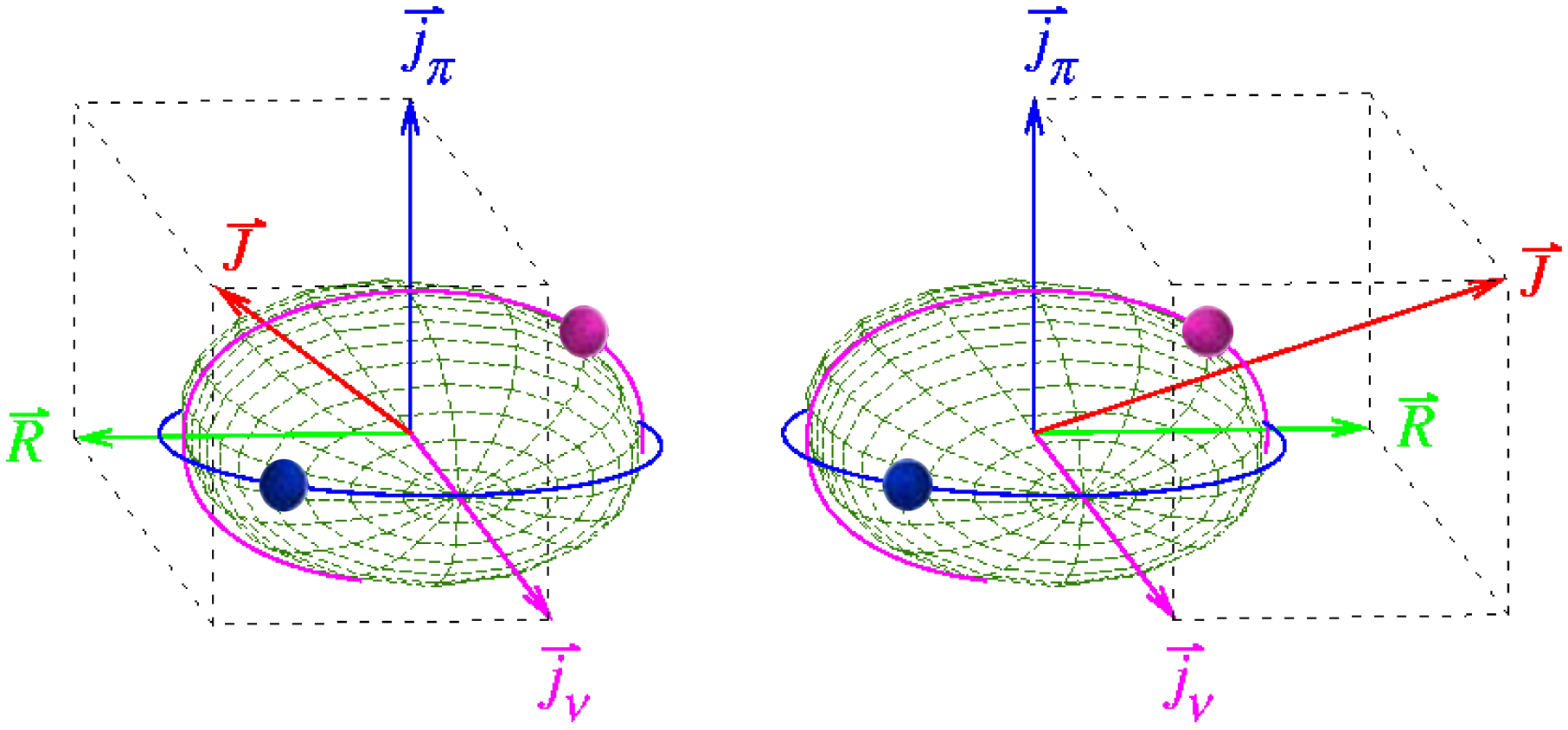
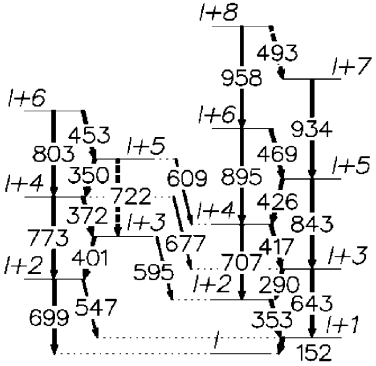
July 27, 2016

Chirality in atomic nuclei

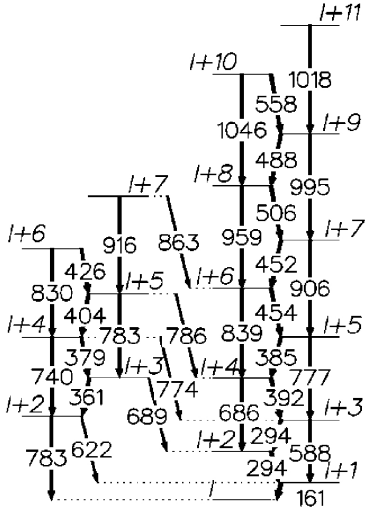
Proposed by S. Frauendorf and J. Meng in Nucl. Phys. **A617**, 131 (1997).



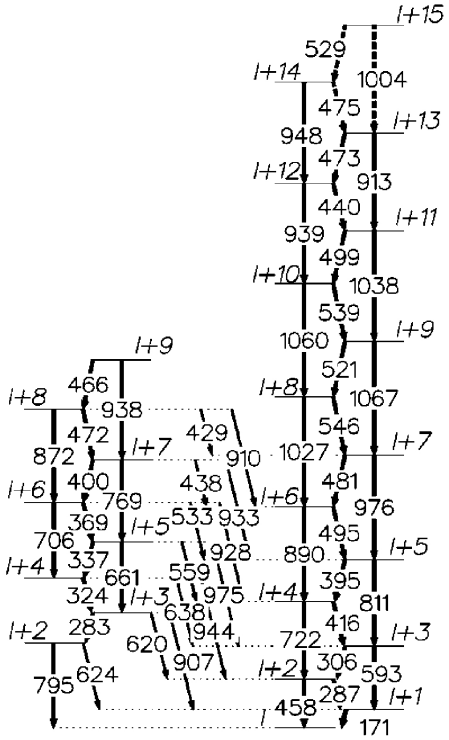
Early experimental searches



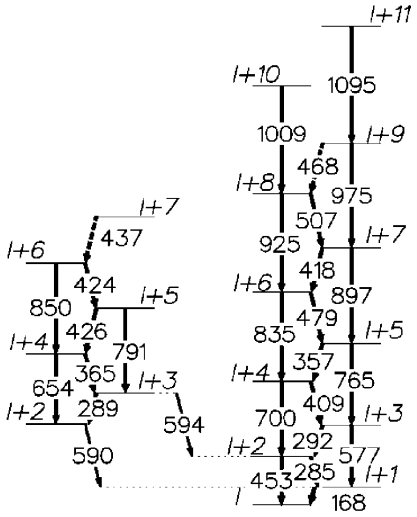
^{130}Cs



^{132}La

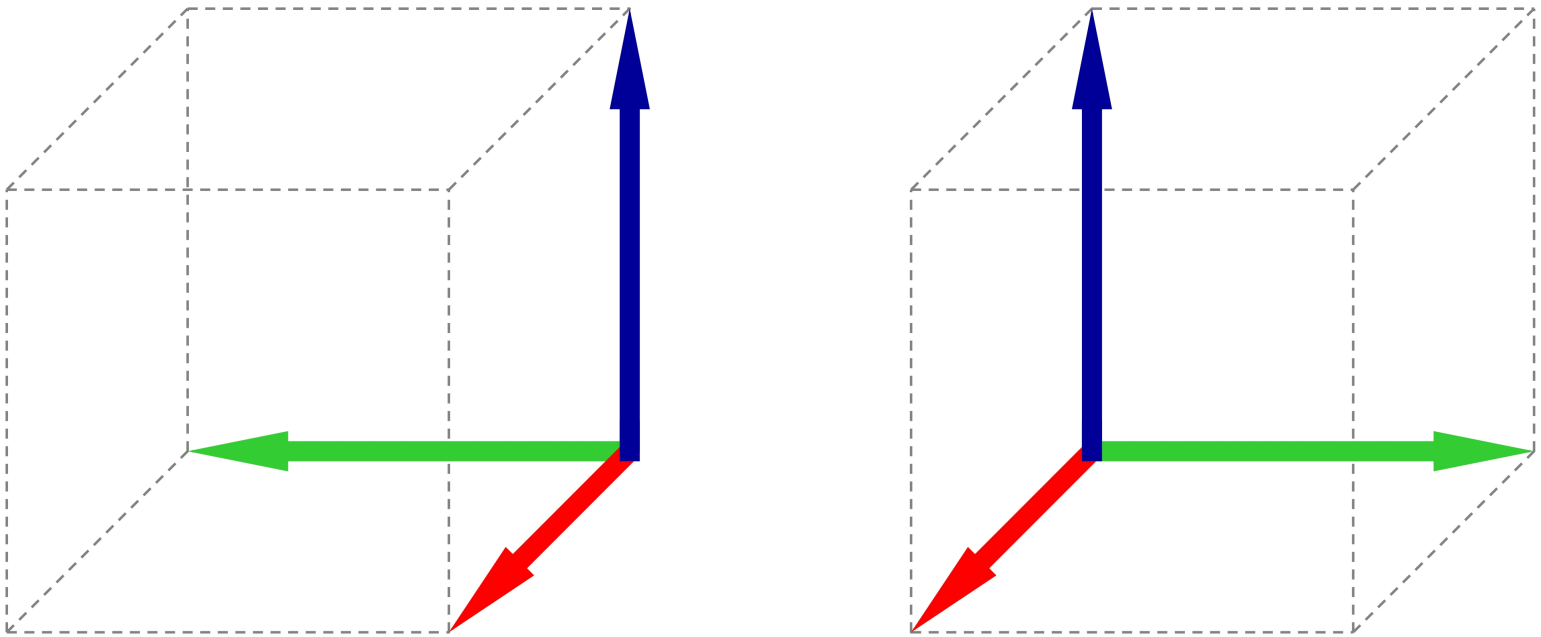


^{134}Pr

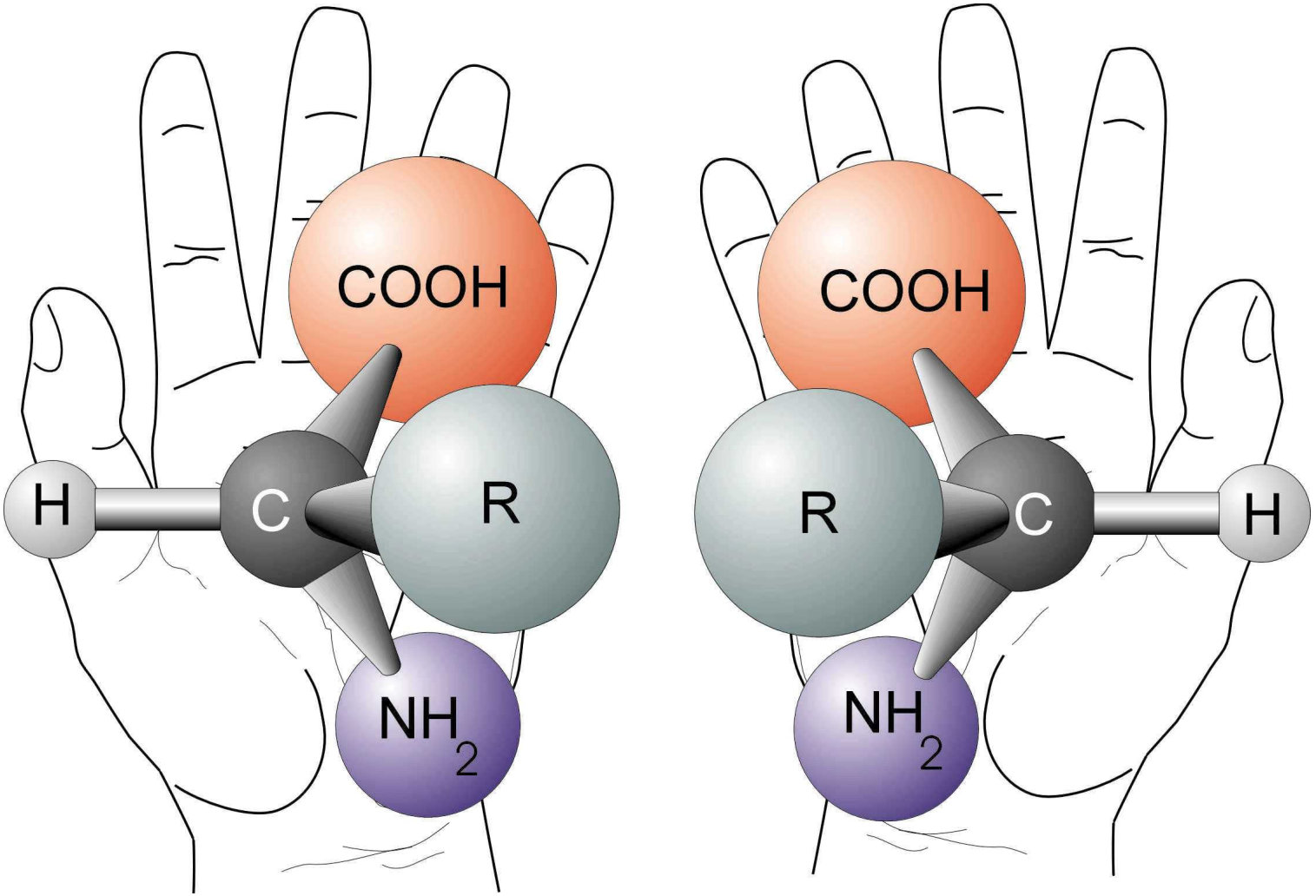


^{136}Pm

Orthogonal coupling of three vectors

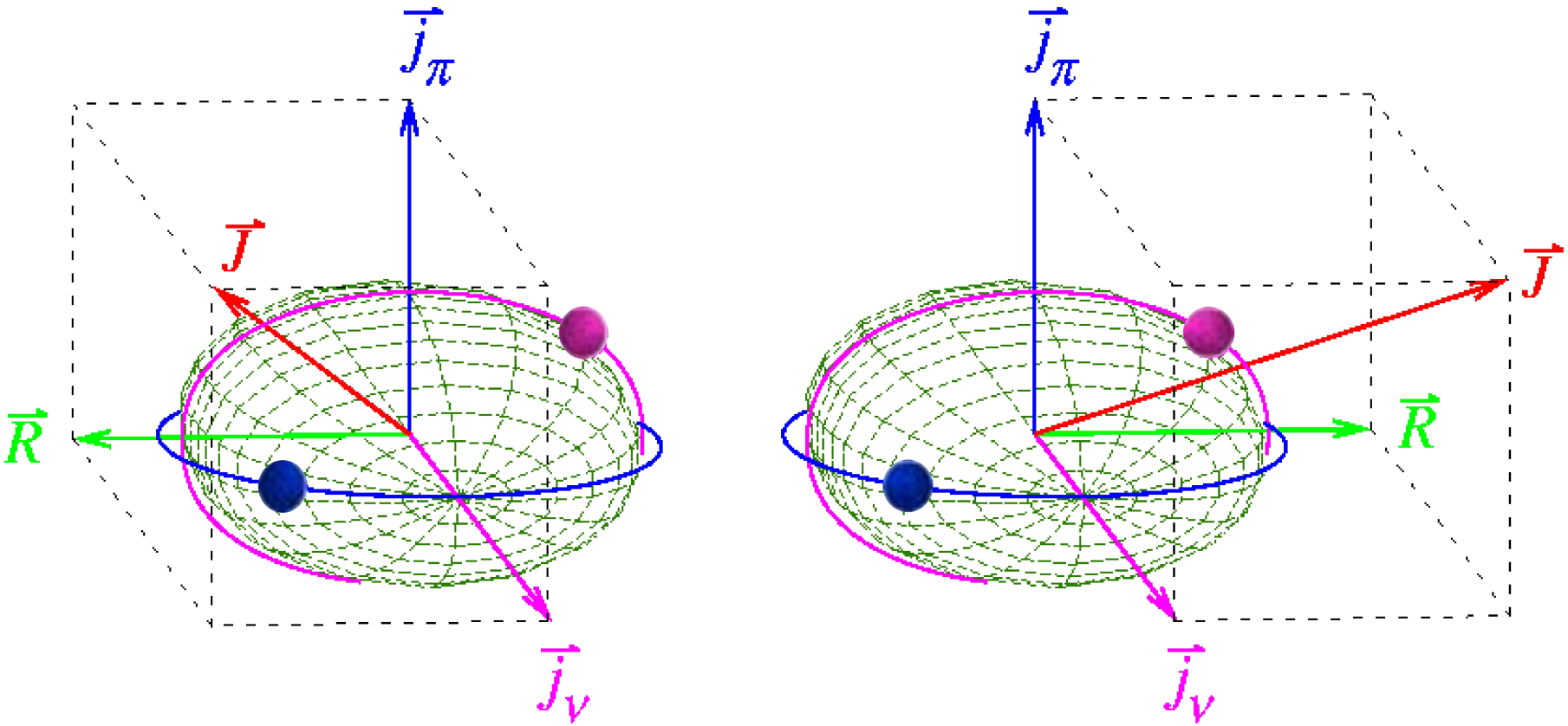


Molecular (geometric) chirality

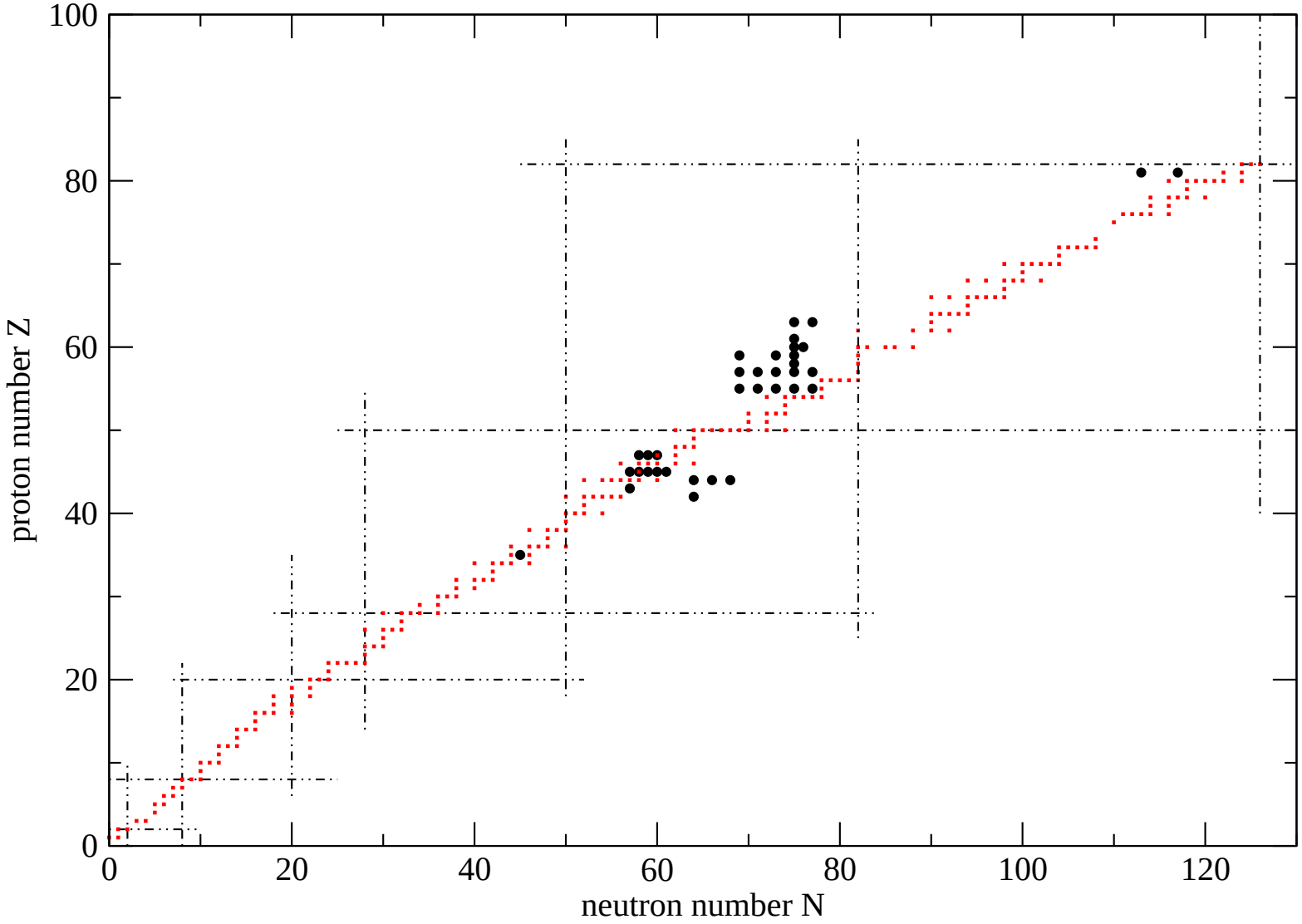


Chirality unique to atomic nuclei

Atomic nuclei are the only systems which can provide two single-particle angular momenta components needed for angular-momentum chirality.

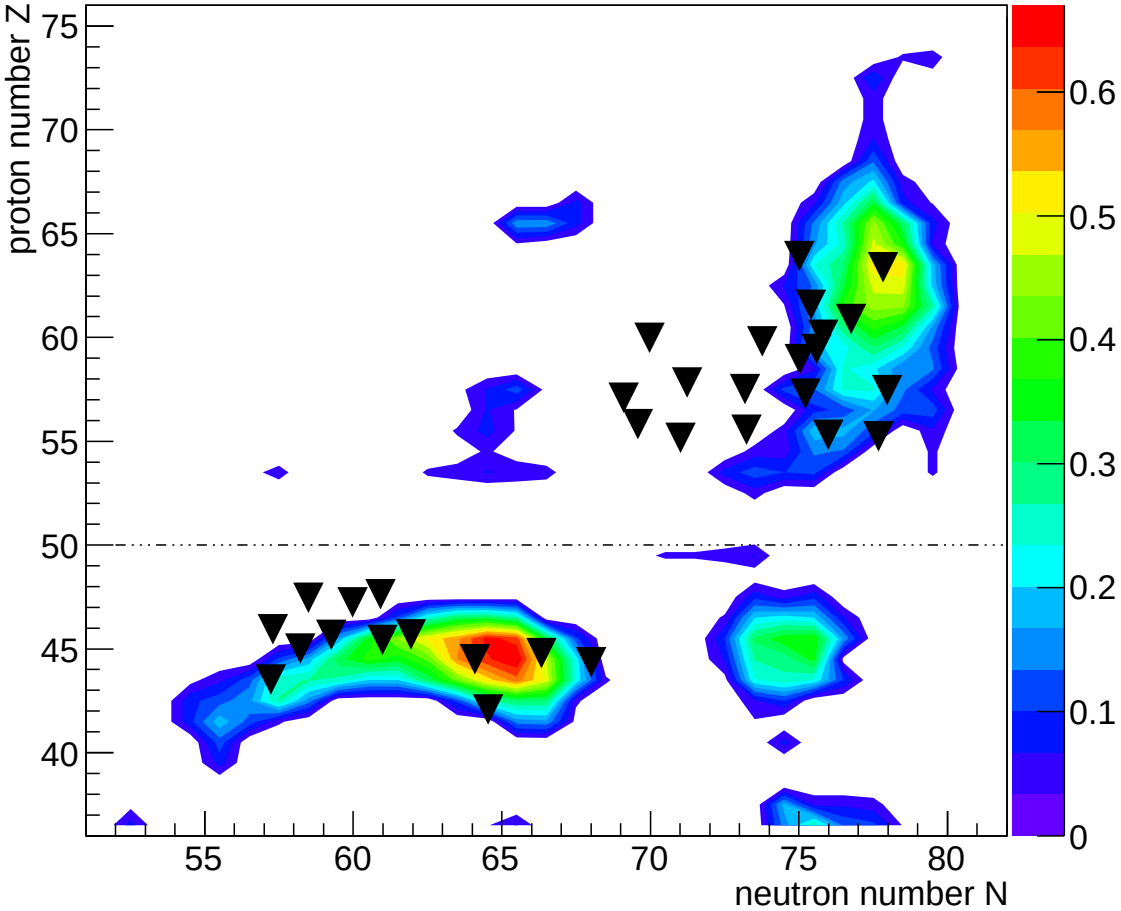


Doublet bands in triaxial regions



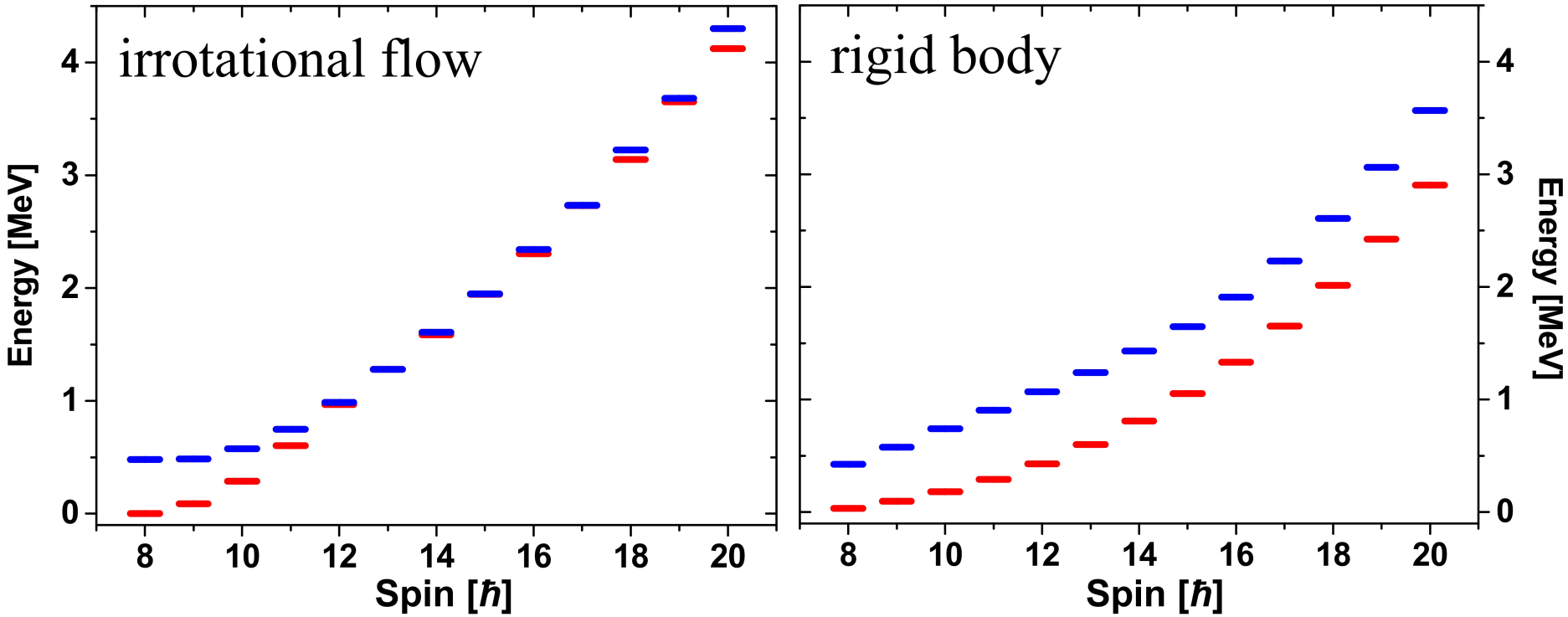
Doublet bands in triaxial regions

Colour denotes energy gain resulting from activating the triaxial degrees of freedom in the Möller-Nix mass model.



Signature of irrotational flow

$$J_k^{irr.} = J_0 \frac{4}{3} \sin^2(\gamma - \frac{2}{3}\pi k) \qquad J_k^{rig.} = J_0 (1 + \sqrt{\frac{5}{4\pi}} \beta \cos(\gamma - \frac{2}{3}\pi k))$$



Is there a smoking gun?

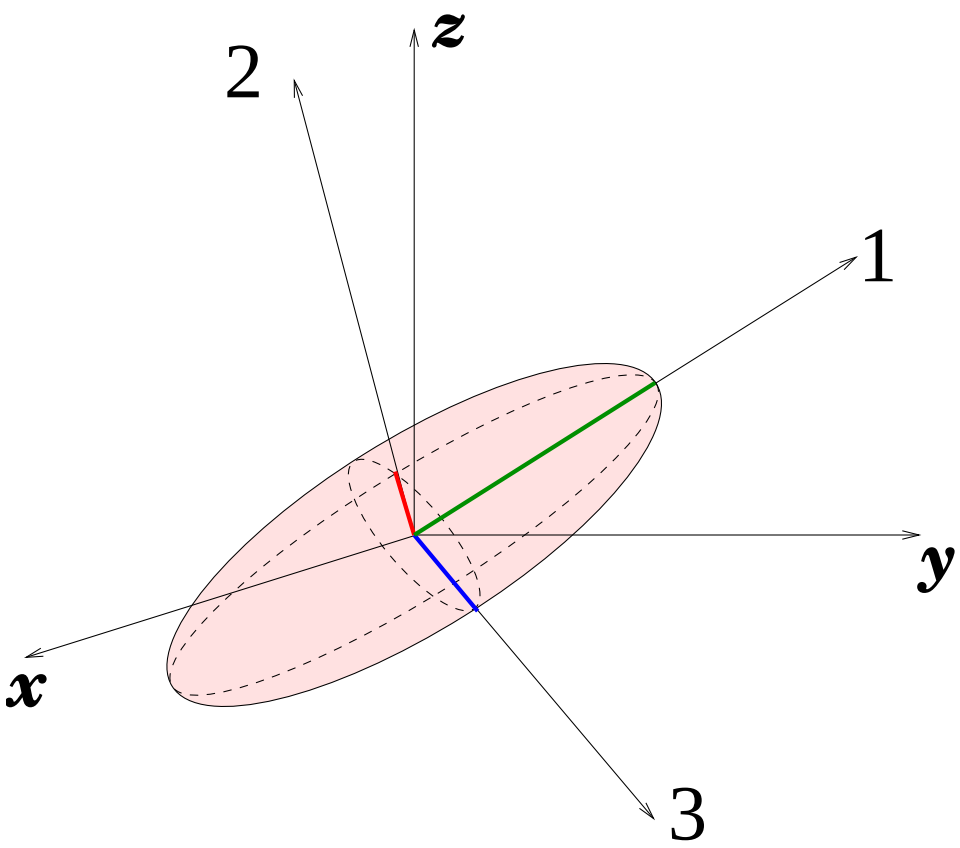
Is there a smoking gun?

- For cases with the smallest energy separation between doublet states, ^{134}Pr and ^{104}Rh both with $\Delta E < 50$ keV, inconsistent behaviour of the electromagnetic properties is a problem.
- In other cases, if the transition rates are all right, the degeneracy of levels is not that great.
- There is no single case which would show in a satisfactory way all properties expected of nuclear chirality.
- Various scenarios are used to explain this fact, most notably, tunnelling through the barrier enabling chiral vibrations of the total angular momentum.

Back to the basics

- Despite ∼20 years of research chirality is extracted from expectation values of orientation operators, rather than being a starting point in construction of nuclear models.
- In that sense, chirality has been perceived as an approximate symmetry attained only in a limited range of angular momentum.
- However, using the particle-hole-coupling model for triaxial odd-odd nuclei it is possible to construct a basis which contains right-handed, left-handed, and planar states of angular momentum coupling.
- If this basis is used, the chirality is explicit rather than extracted feature as in any other models with non-chiral basis.

Laboratory/intrinsic reference frame



$$|RM\rangle = \sum_{K=-R}^{K=R} D^R_{MK}(\omega)\,|RK\rangle$$

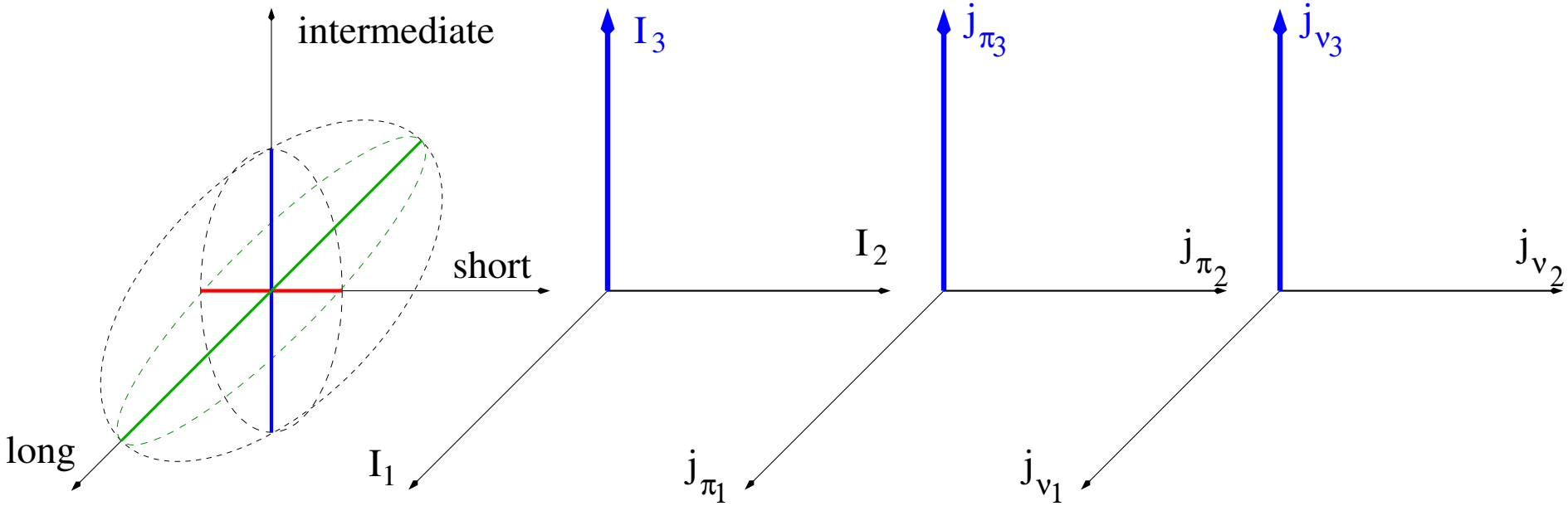
R angular momentum

M z-projection of R

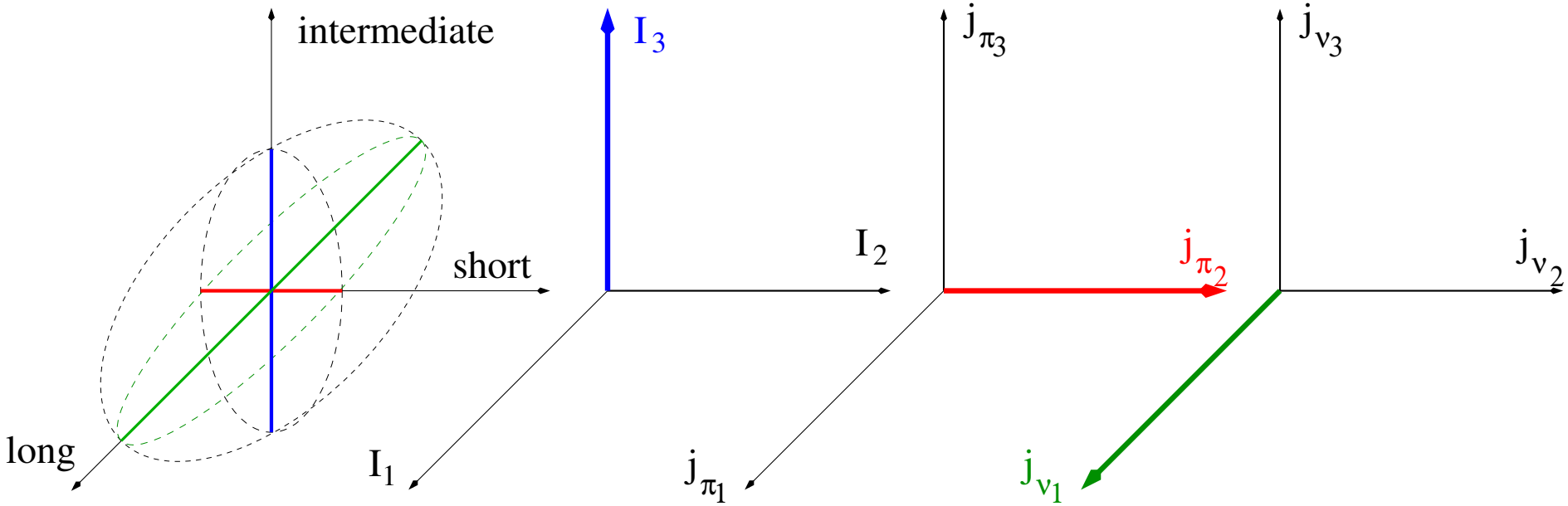
K 3-projection of R

$\omega = (\phi, \theta, \psi)$ Euler angles

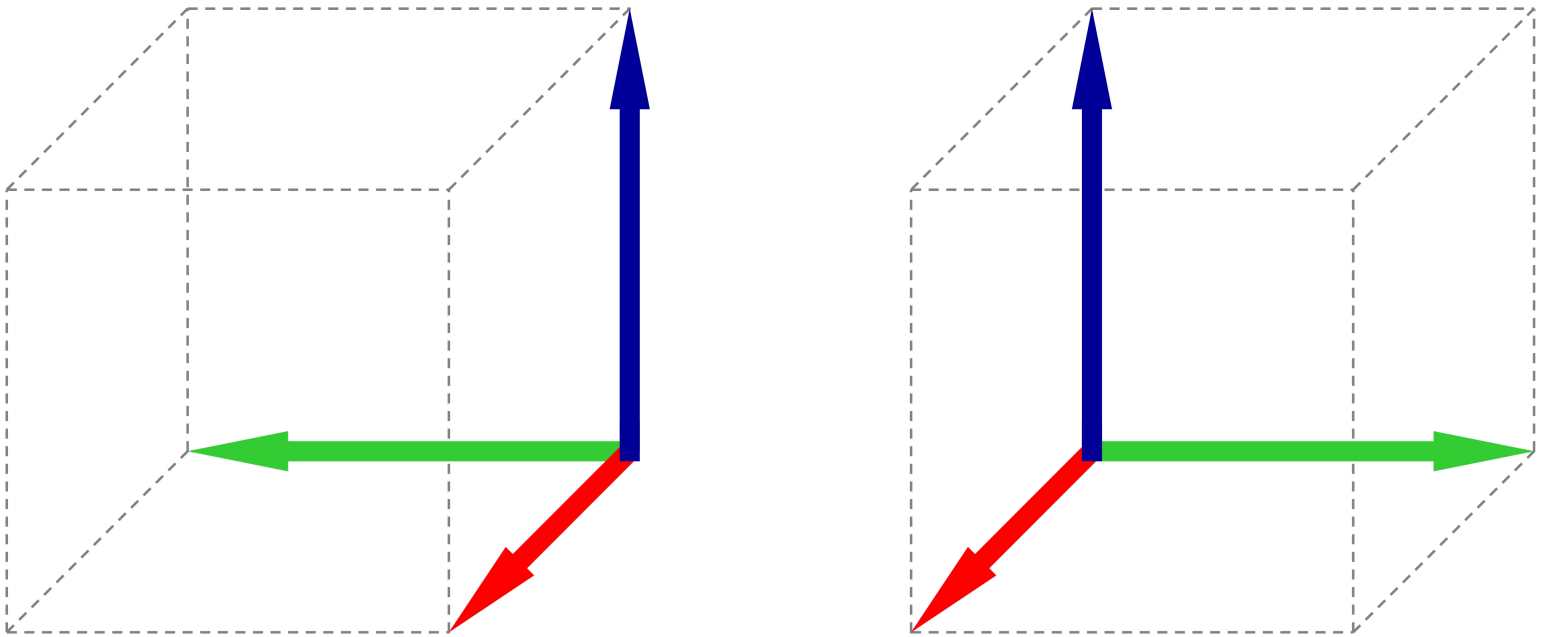
Standard particle-hole-rotor basis



Chiral particle-hole-rotor basis



Orthogonal coupling of three vectors



Particle-hole-triaxial-rotor model

The chiral basis results from a $\pi/2$ rotation of standard single-particle proton/neutron states around the 1st (long) axis and a $\pi/2$ rotation of standard single-particle neutron/proton states around the 2nd (short) axis

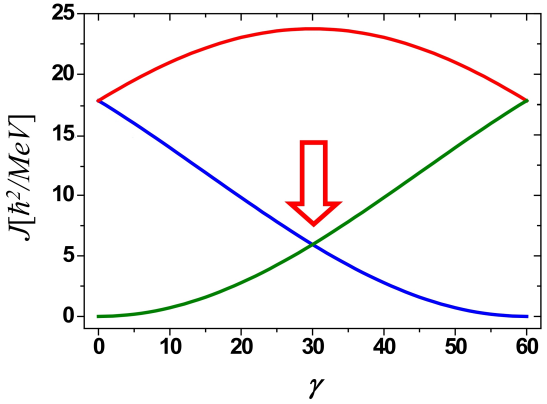
$$|j\kappa\rangle = \sum_{\Omega} d^{j}_{\kappa\Omega}\left(-\pi/2\right)|j\Omega\rangle$$

$$|j\varkappa\rangle = \sum_{\Omega}(-i)^{\varkappa-\Omega} d^{j}_{\varkappa\Omega}\left(-\pi/2\right)|j\Omega\rangle$$

$\kappa/\varkappa$ projection of j_ν/j_π on the long/short axis.

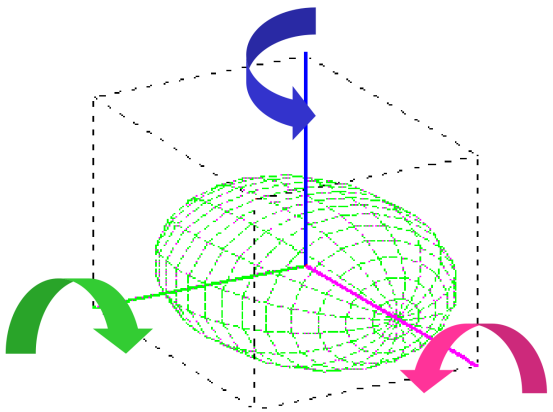
Correlations built into the basis prior to diagonalization.

Axial rotation of triaxial body



$$
\begin{aligned}
J_1 &= J_2 = \frac{1}{4}J_0, \quad J_3 = J_0 \\
H_R &= \sum_{k=1}^{3} \frac{R_k^2\hbar^2}{2J_k} = \frac{2\hbar^2}{J_0}((R_1^2 + R_2^2) + \frac{1}{4}R_3^2) = \\
& \frac{2\hbar^2}{J_0}((R^2 - R_3^2) + \frac{1}{4}R_3^2) = \frac{2\hbar^2}{J_0}(R^2 - \frac{3}{4}R_3^2) \\
E_{RK} &= \frac{2\hbar^2}{J_0}\left(R(R+1) - \frac{3}{4}K^2\right)
\end{aligned}
$$

D_2 symmetry of the Hamiltonian



$$
\begin{aligned}
\mathcal{R}_\kappa^2(\pi) &= \mathbb{1}, \quad \kappa = 1,2,3 \\
\left[R_k^2, \mathcal{R}_\kappa(\pi)\right] &= 0, \quad k = 1,2,3, \quad \kappa = 1,2,3 \\
\left[H_R, \mathcal{R}_\kappa(\pi)\right] &= 0, \quad \kappa = 1,2,3
\end{aligned}
$$

Single-j approximation in the deformed mean field

$$
\begin{aligned}
j_0 &= j_3, \\
j_{\pm 1} &= \mp \frac{1}{\sqrt{2}} \left(j_1 \pm i j_2 \right) \\
\left\langle j, \Omega \middle| Y_{2,0} \middle| j, \Omega' \right\rangle &= -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \frac{\left\langle j, \Omega \middle| 3 j_0^2 - j^2 \middle| j, \Omega' \right\rangle}{j(j+1)} \\
\left\langle j, \Omega \middle| Y_{2,\pm 2} \middle| j, \Omega' \right\rangle &= -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \sqrt{3} \frac{\left\langle j, \Omega \middle| j_{\pm 1}^2 \middle| j, \Omega' \right\rangle}{j(j+1)}
\end{aligned}
$$

Particle-hole-triaxial-rotor model chiral wave functions

$$
\begin{aligned}
|IMKj_\pi \varkappa_\pi j_\nu \kappa_\nu\rangle &= \frac{1}{2}\sqrt{\frac{2I+1}{8\pi^2}}\Big(D^I_{MK}(\omega)\,|j_\pi \varkappa_\pi\rangle\,|j_\nu \kappa_\nu\rangle \\
&+ (-1)^{I+j_\pi+\kappa_\nu} D^I_{M\overline{K}}(\omega)\,|j_\pi \overline{\varkappa}_\pi\rangle\,|j_\nu \kappa_\nu\rangle \\
&+ (-1)^{I-j_\nu+K-\varkappa_\pi-\kappa_\nu} D^I_{M\overline{K}}(\omega)\,|j_\pi \varkappa_\pi\rangle\,|j_\nu \overline{\kappa}_\nu\rangle \\
&+ (-1)^{j_\pi+j_\nu+K-\varkappa_\pi} D^I_{MK}(\omega)\,|j_\pi \overline{\varkappa}_\pi\rangle\,|j_\nu \overline{\kappa}_\nu\rangle\Big)
\end{aligned}
$$

Particle-hole-triaxial-rotor model chiral wave functions

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&+ (-1)^{j_\pi+j_\nu+K-\varkappa_\pi} D^I_{MK}(\omega)\,|j_\pi \overline{\varkappa}_\pi\rangle\,|j_\nu \overline{\kappa}_\nu\rangle\Big)
\end{aligned}
$$

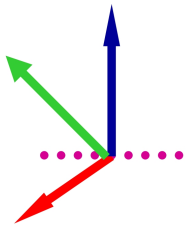
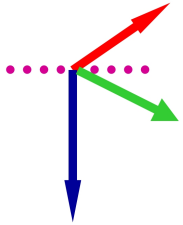
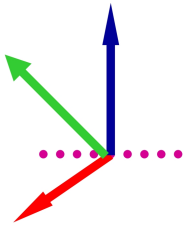
$K = 0$ or $\varkappa = \pm 1/2$ or $\kappa = 1/2$: planar states

$K > 0,\ \varkappa \in [3/2, j]\,, \kappa \in [3/2, j]$: right-handed states

$K > 0,\ \varkappa \in [-j, -3/2]\,, \kappa \in [3/2, j]$: left-handed states

Planar states

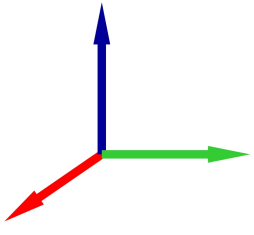
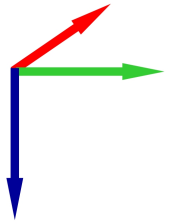
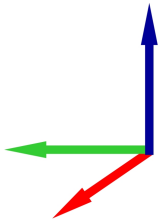
$$O\,|I\,P\rangle = \boldsymbol{T}\,R_y(\pi)\,|I\,P\rangle = |I\,P\rangle\,.$$

$\boldsymbol{T}\,R_y(\pi)\Big|$  $\Big\rangle = \boldsymbol{T}\Big|$  $\Big\rangle = \Big|$  $\Big\rangle$

Chiral states

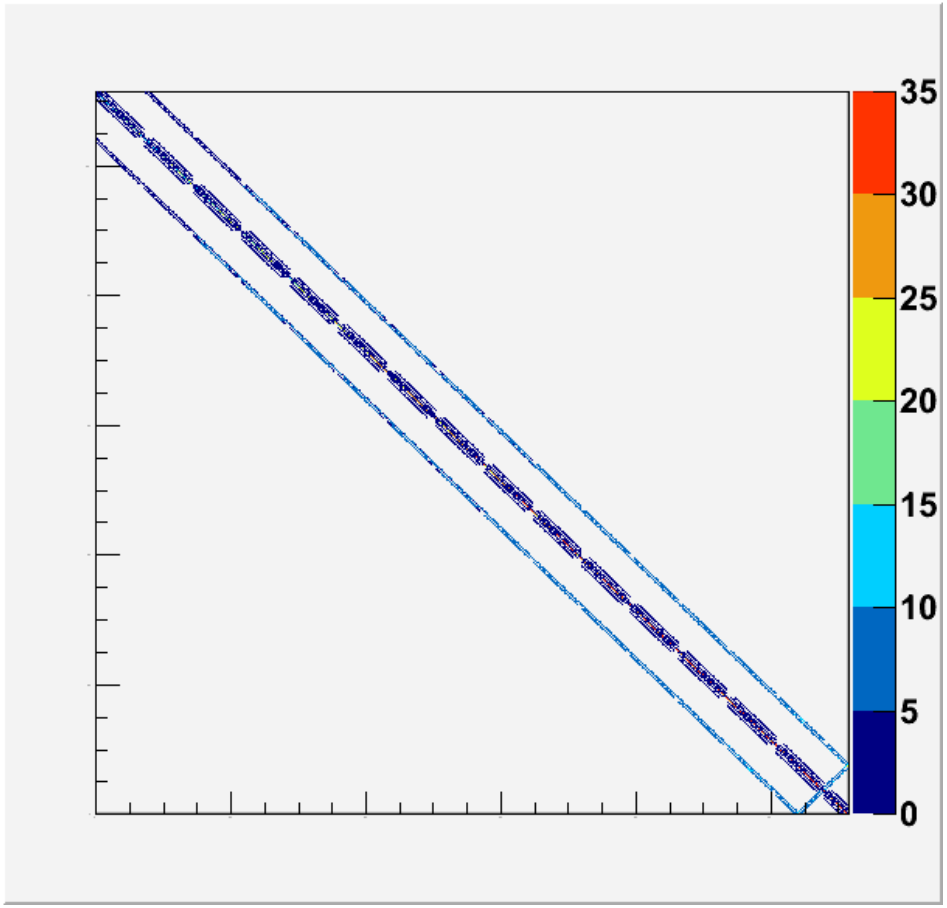
$$O\,|I\,R\rangle = \boldsymbol{T}\,R_y(\pi)\,|I\,R\rangle = |I\,L\rangle\,,$$

$$O\,|I\,L\rangle = \boldsymbol{T}\,R_y(\pi)\,|I\,L\rangle = |I\,R\rangle\,.$$

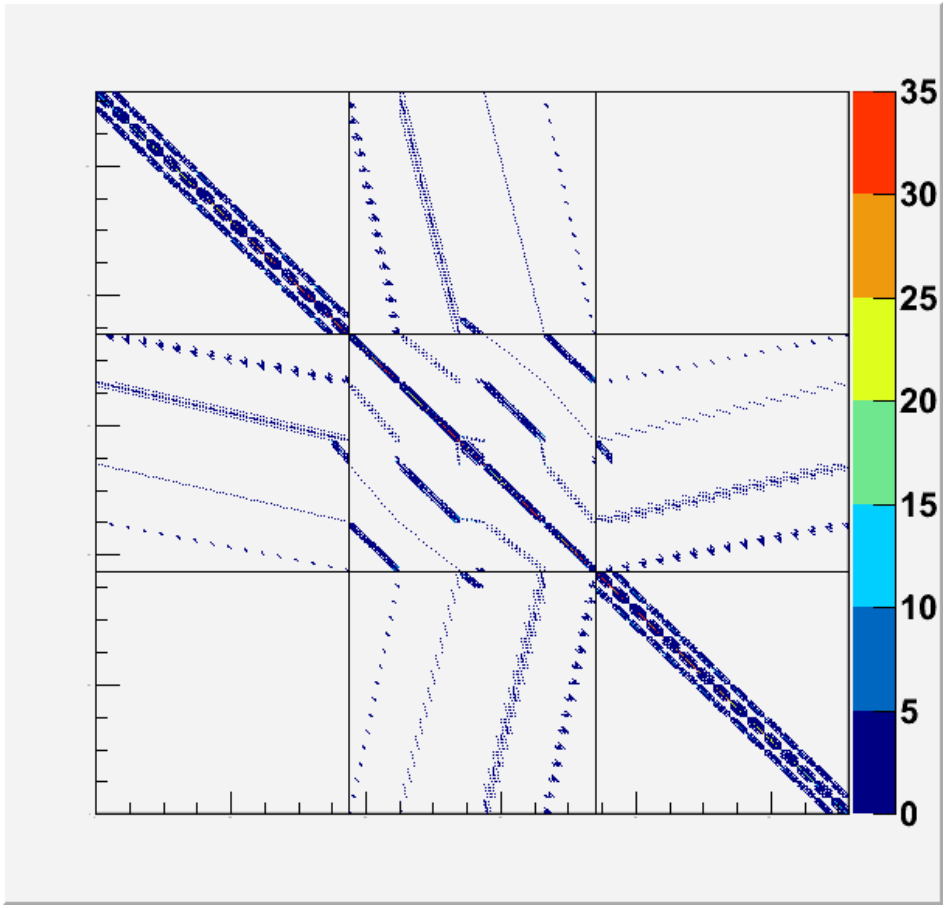
$\boldsymbol{T}\,R_y(\pi)\Big|$  $\Big\rangle = \boldsymbol{T}\Big|$  $\Big\rangle = \Big|$  $\Big\rangle$

The Hamiltonian

standard

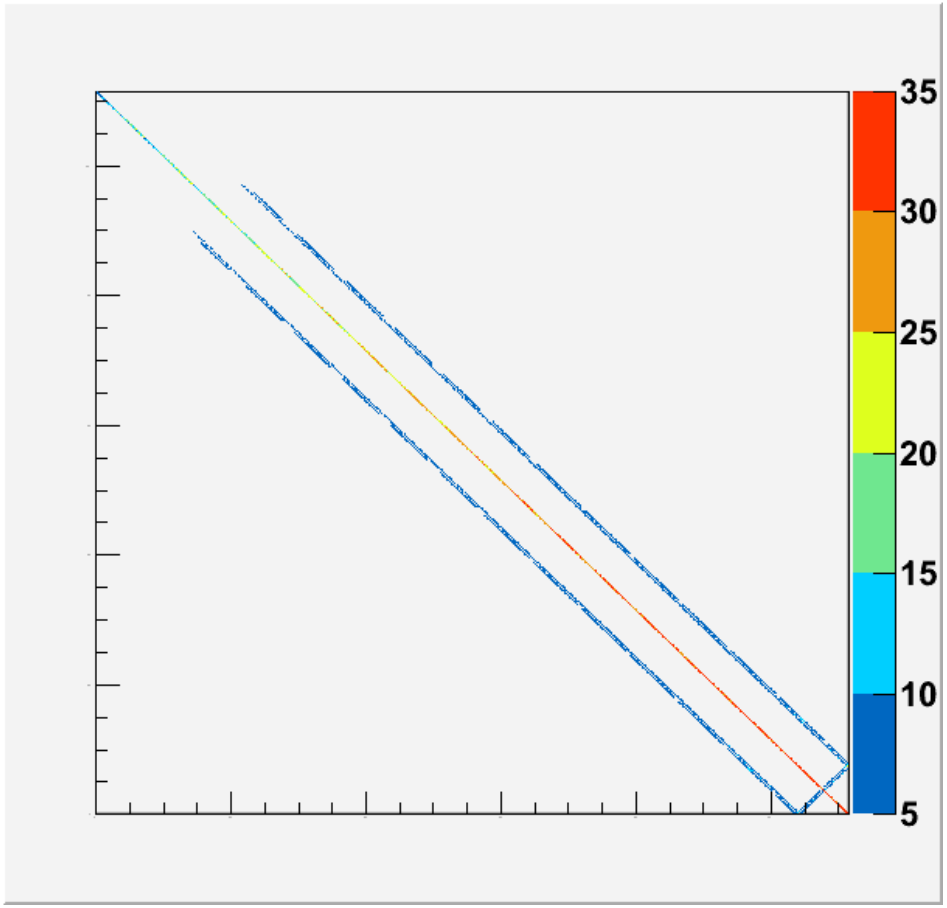


chiral

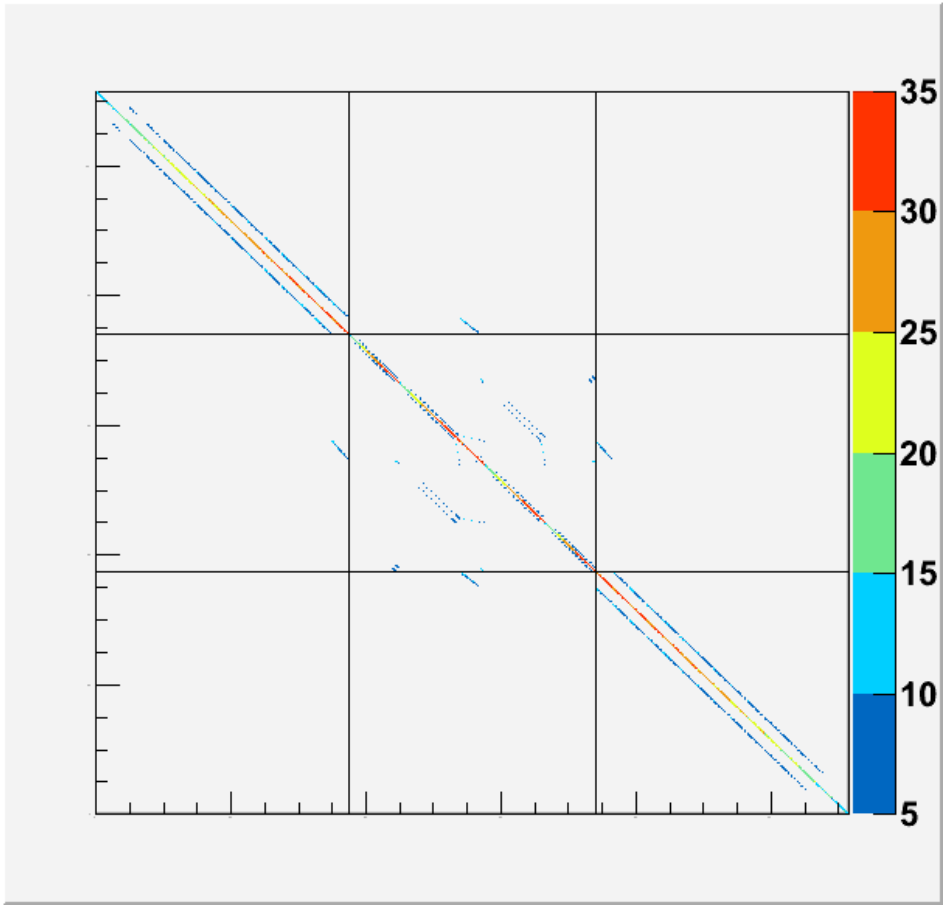


The Hamiltonian

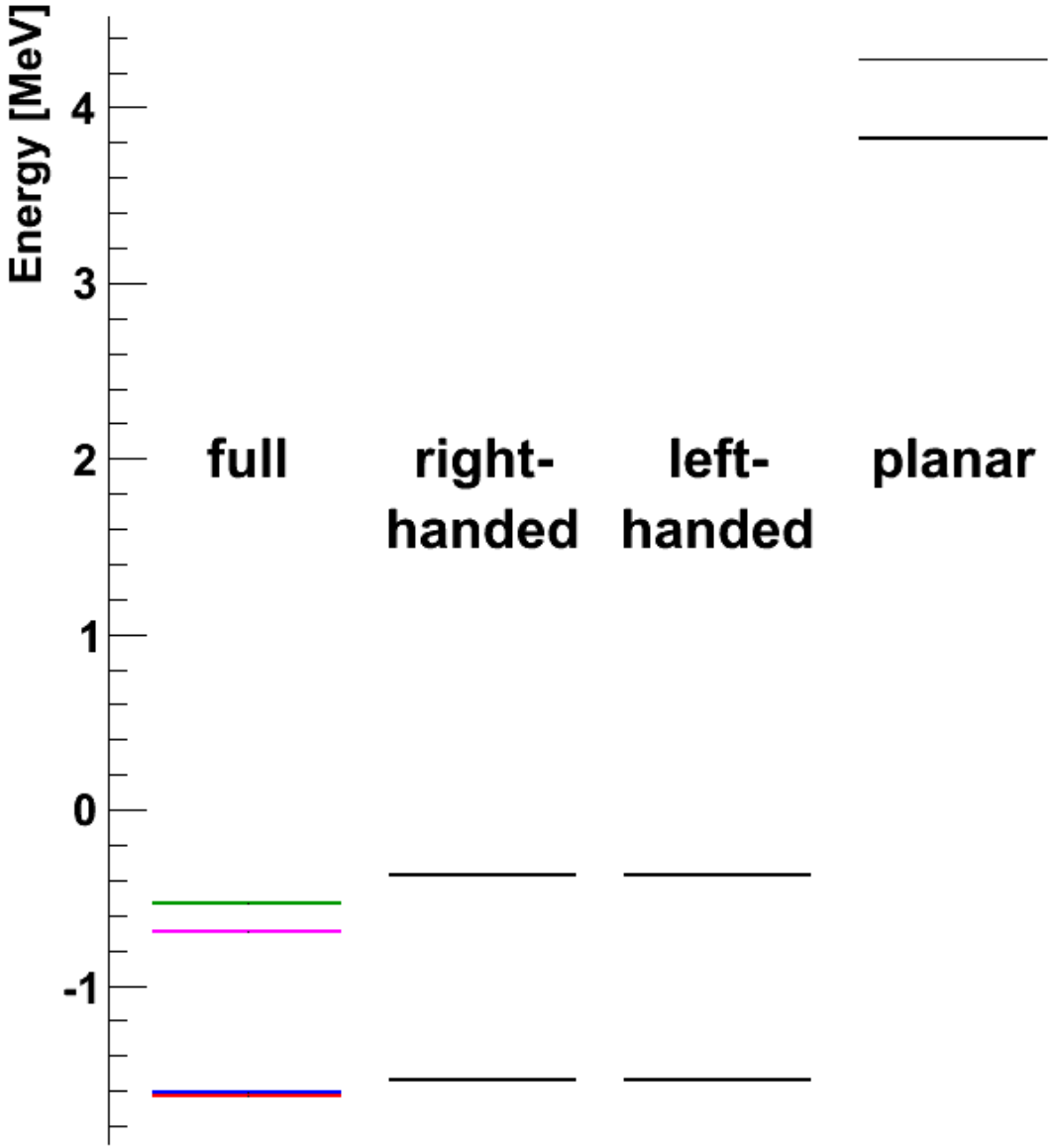
standard



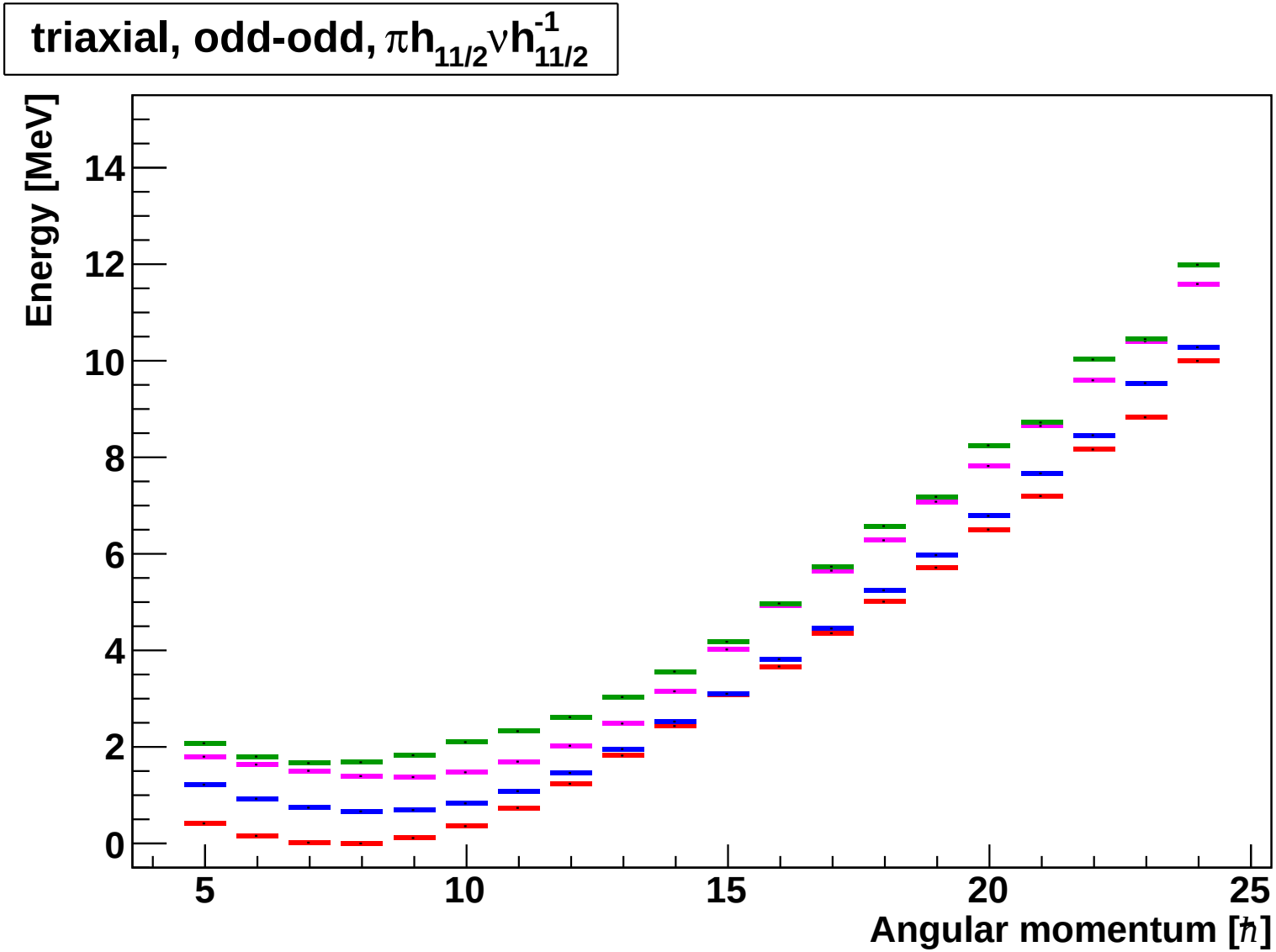
chiral



Eigen states of the Hamiltonian



$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model



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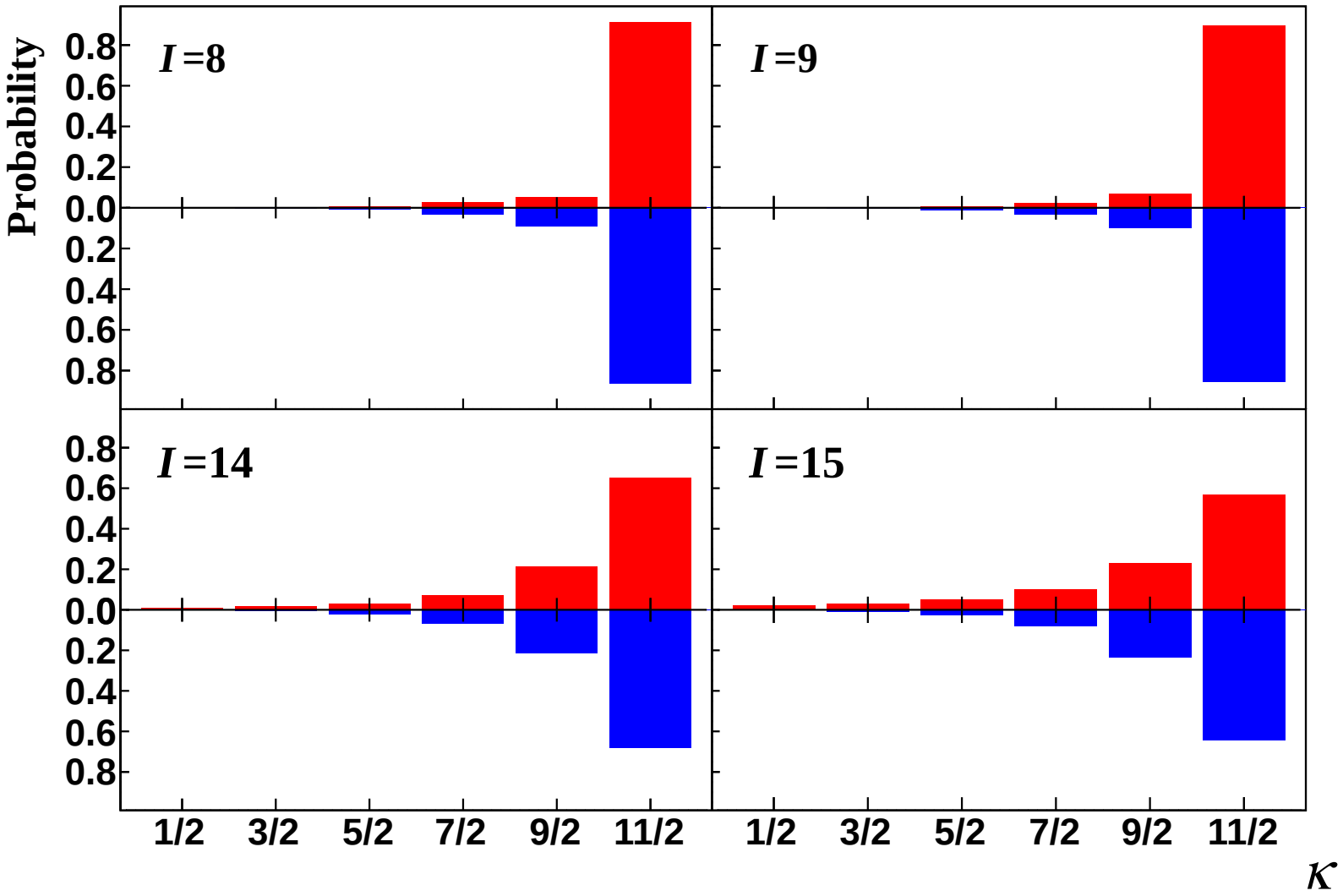

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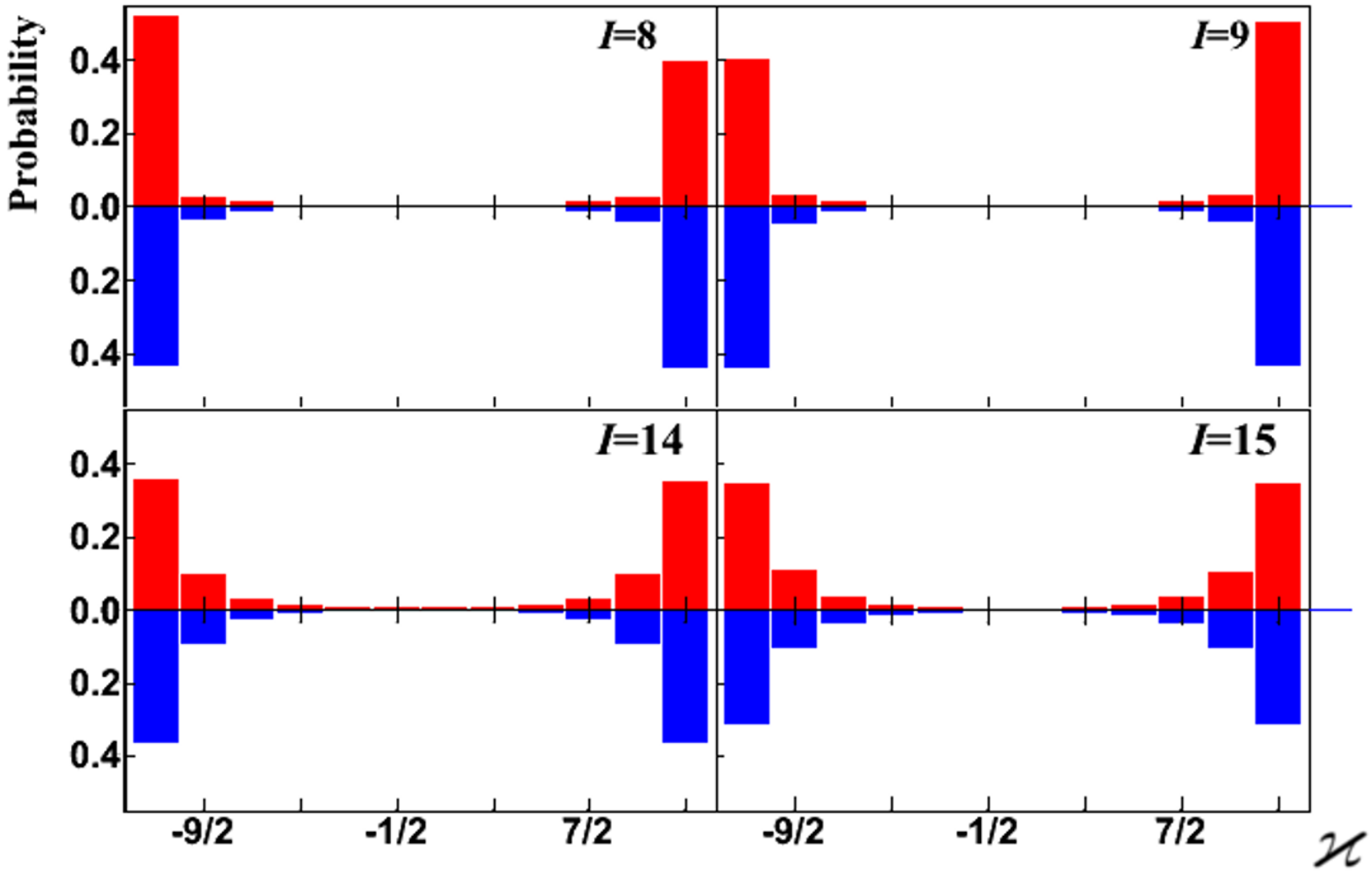

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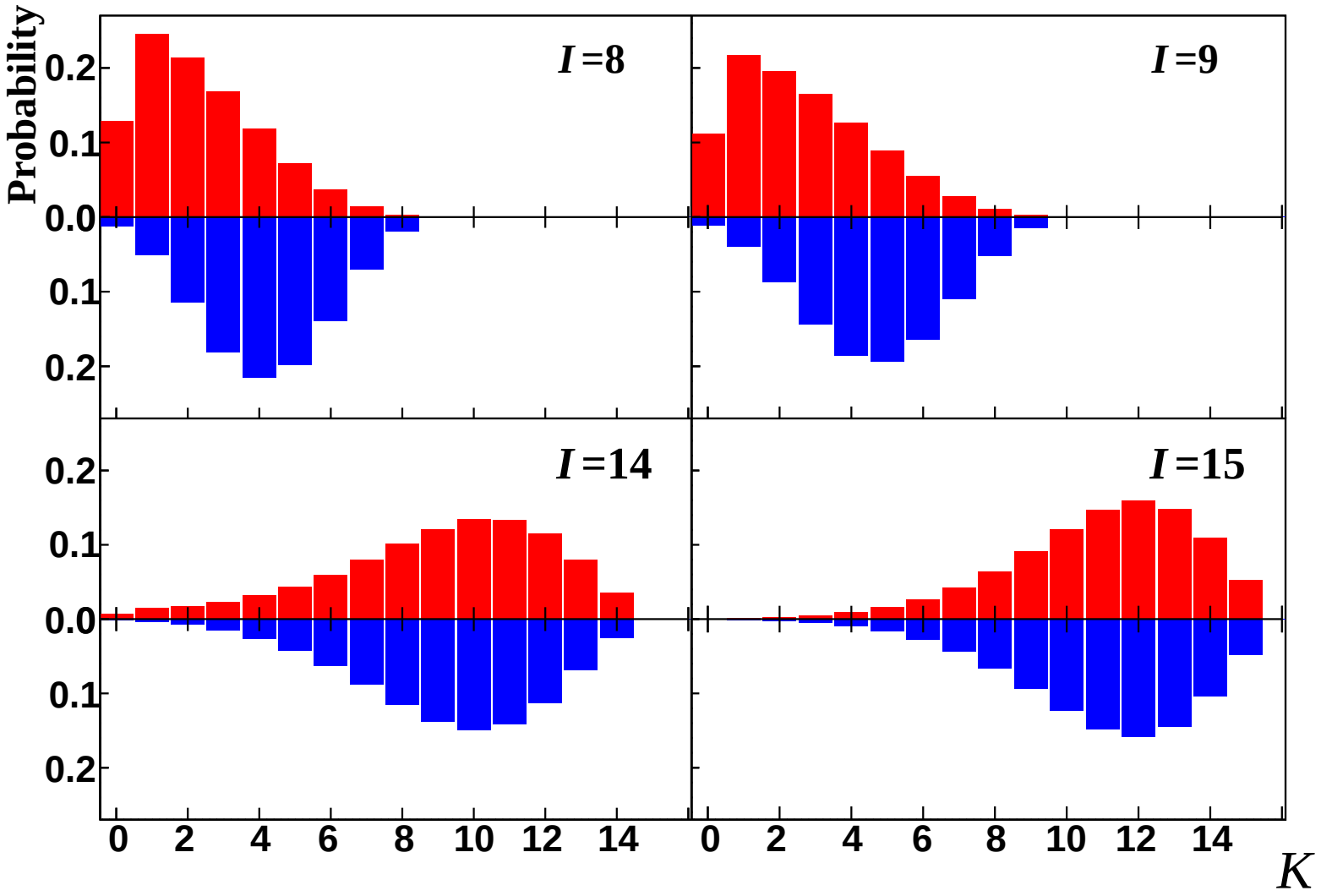


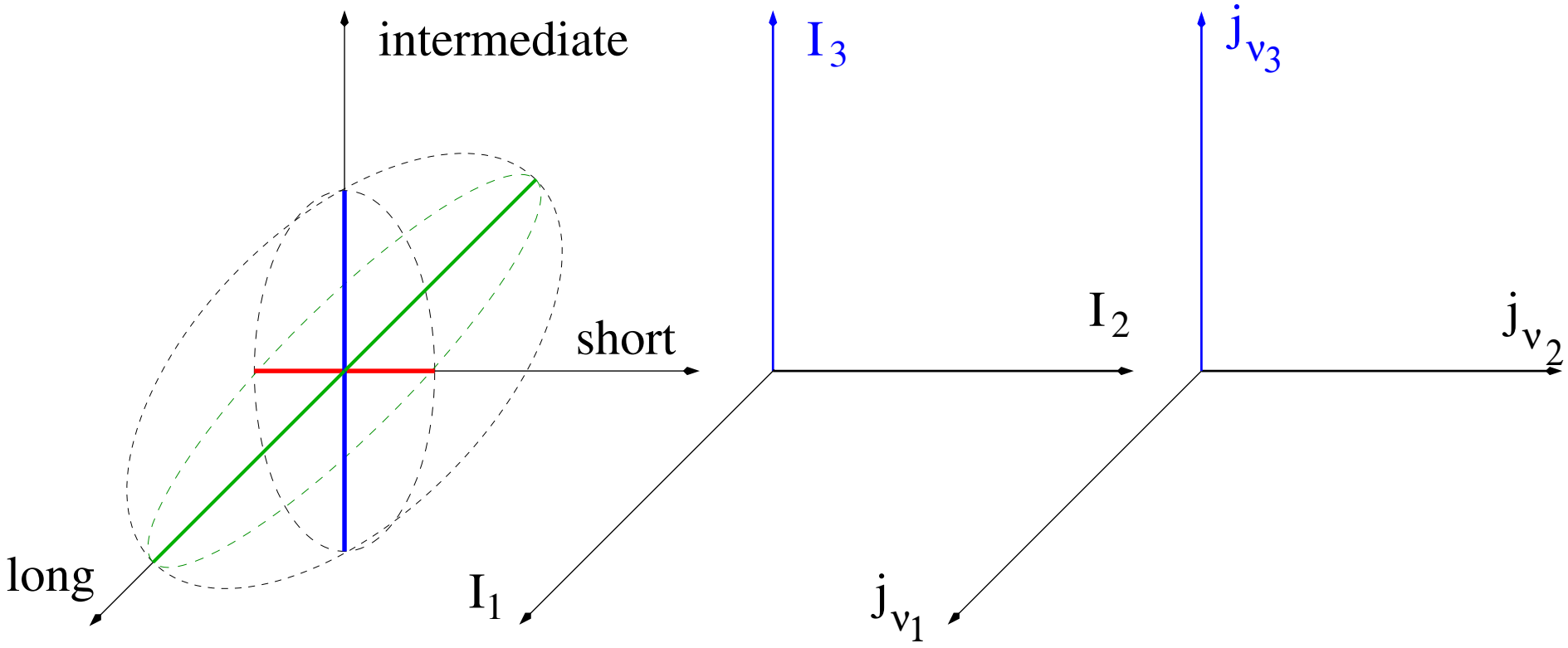
Conclusions

- A new basis is proposed for the triaxial particle-rotor model.
- It is applicable to particle/hole coupling in odd- and odd-odd nuclei.
- High-j particle and high-j hole coupling to the triaxial core results in basis states which are left-handed, right-handed, and planar.
- Chiral basis diagonalize a significant fraction of the model Hamiltonian providing efficient expansion for wave functions of final states.
- Calculations confirm doubling of states arising from orthogonal coupling of angular momentum vectors.
- The wave function expanded in the left-handed, right-handed, and planar states will be used to identify and investigate observables sensitive to chirality in angular momentum coupling.

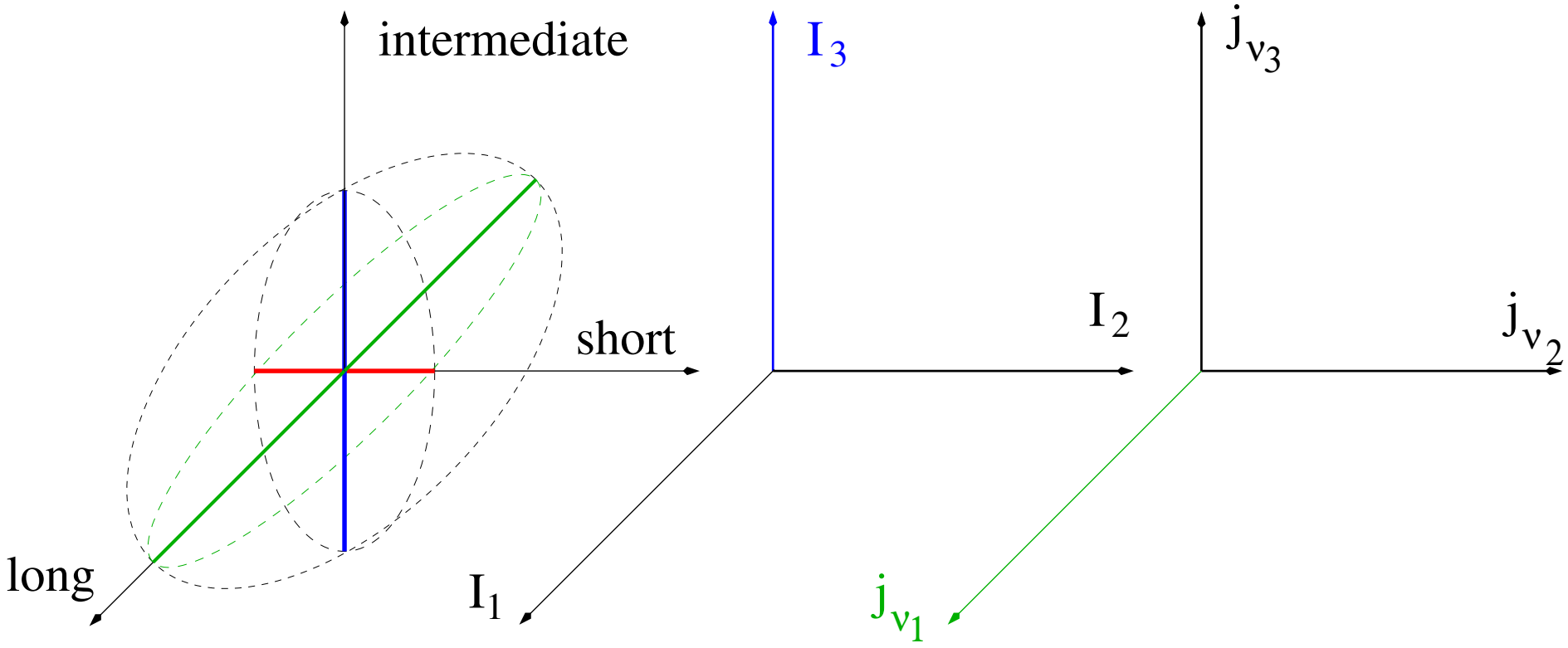
Is there a smoking gun?

The Truth Is Out There.

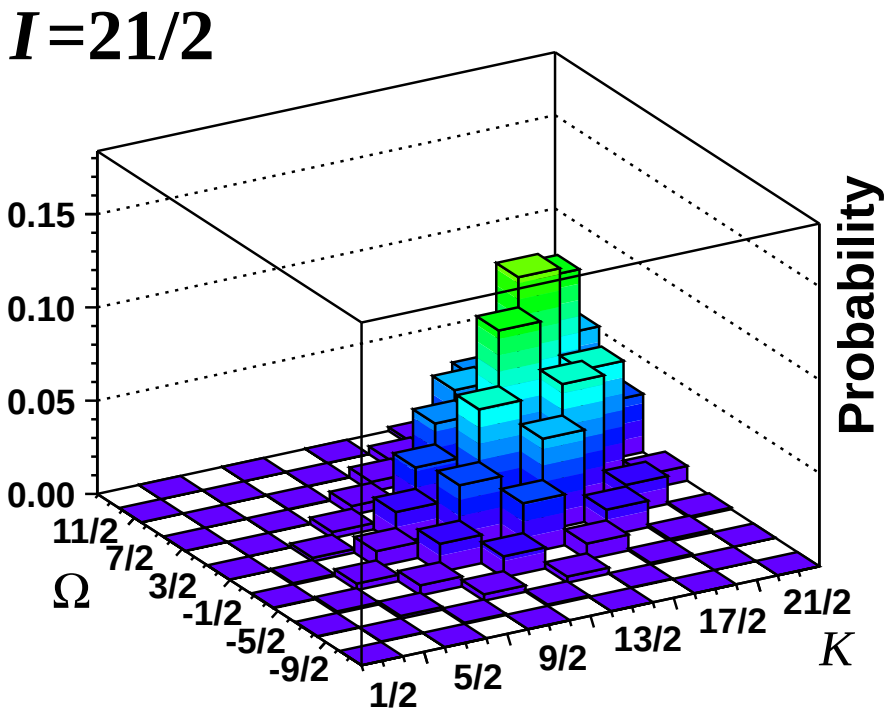
Standard particle-rotor basis



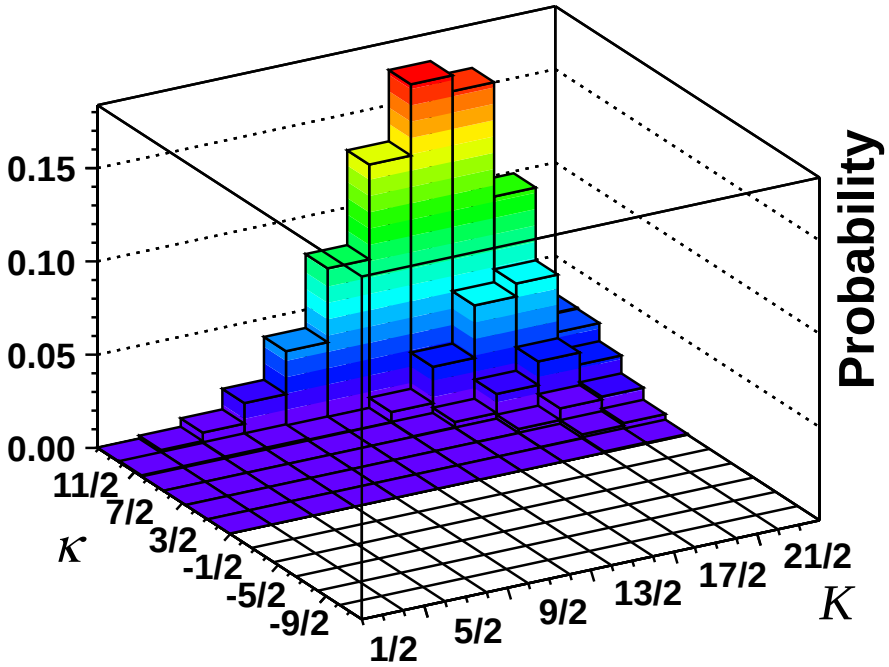
New ("chiral") particle-rotor basis



Wave function in the standard and in the chiral basis

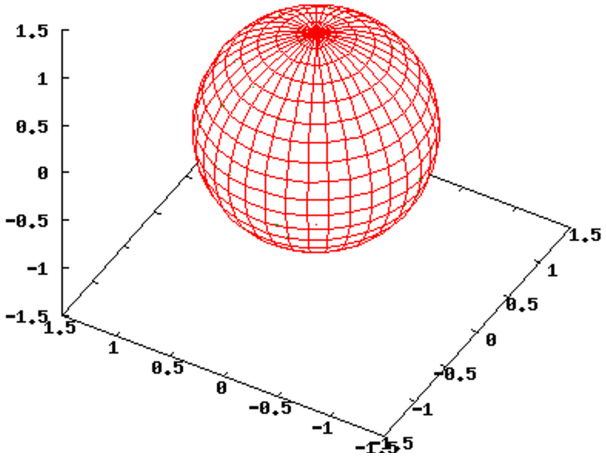


standard

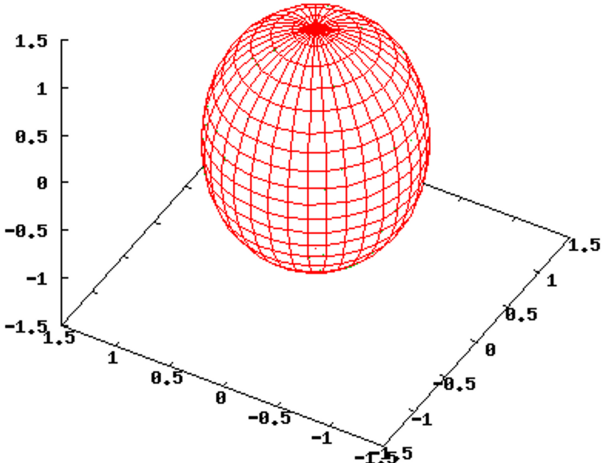


chiral

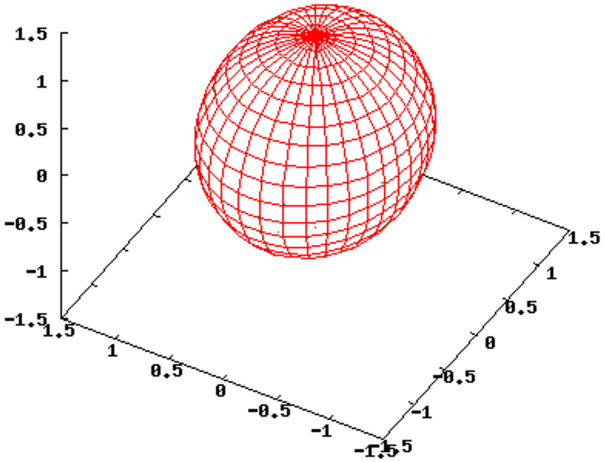
Quadrupole deformation



spherical



axial



triaxial

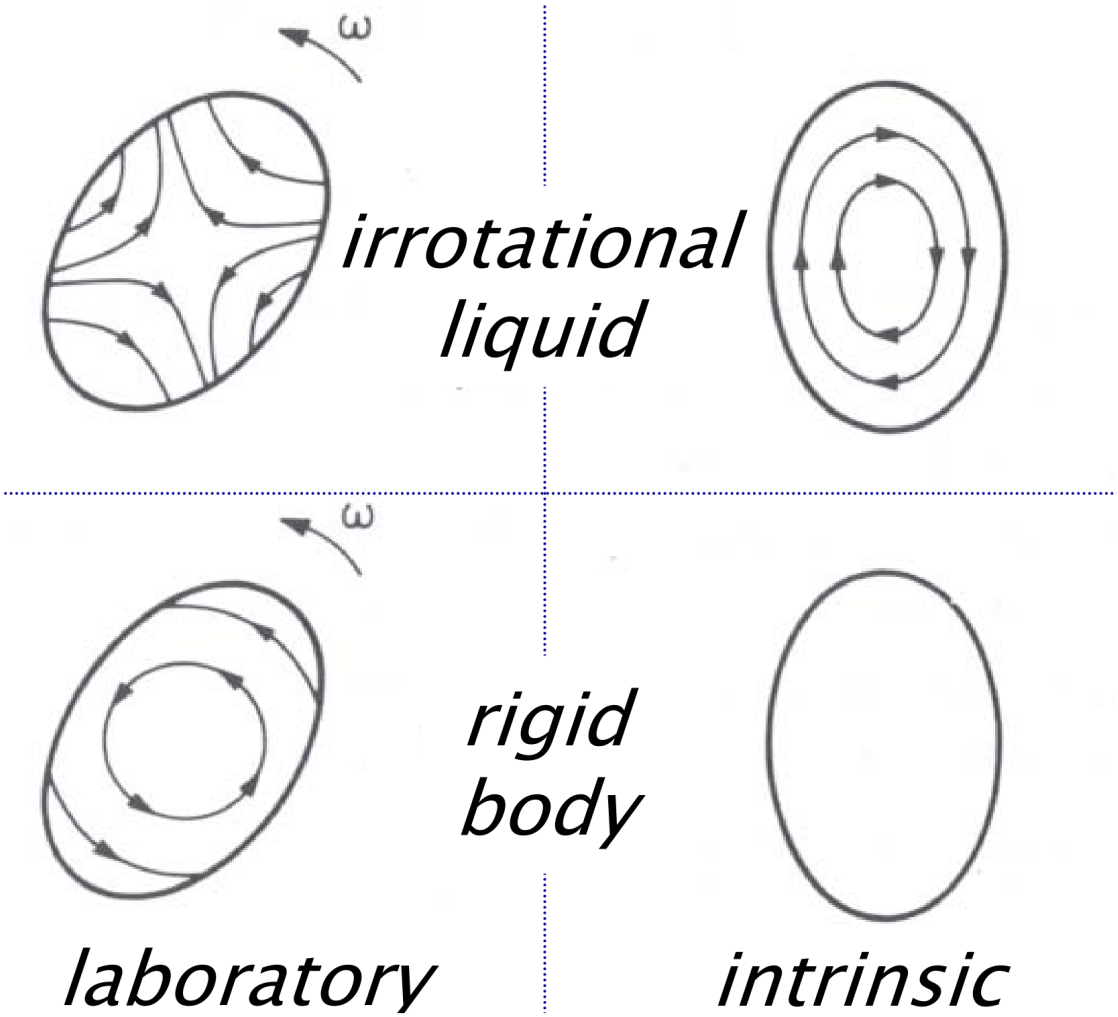
Hamiltonian for a rotor

$$H_R = \frac{\hbar^2}{2}\left(\frac{R_1^2}{J_1} + \frac{R_2^2}{J_2} + \frac{R_3^2}{J_3}\right)$$

R_i for i=1, 2, 3 are rotor's angular momentum components

J_i for i=1, 2, 3 are rotor's principle-axes moments of inertia

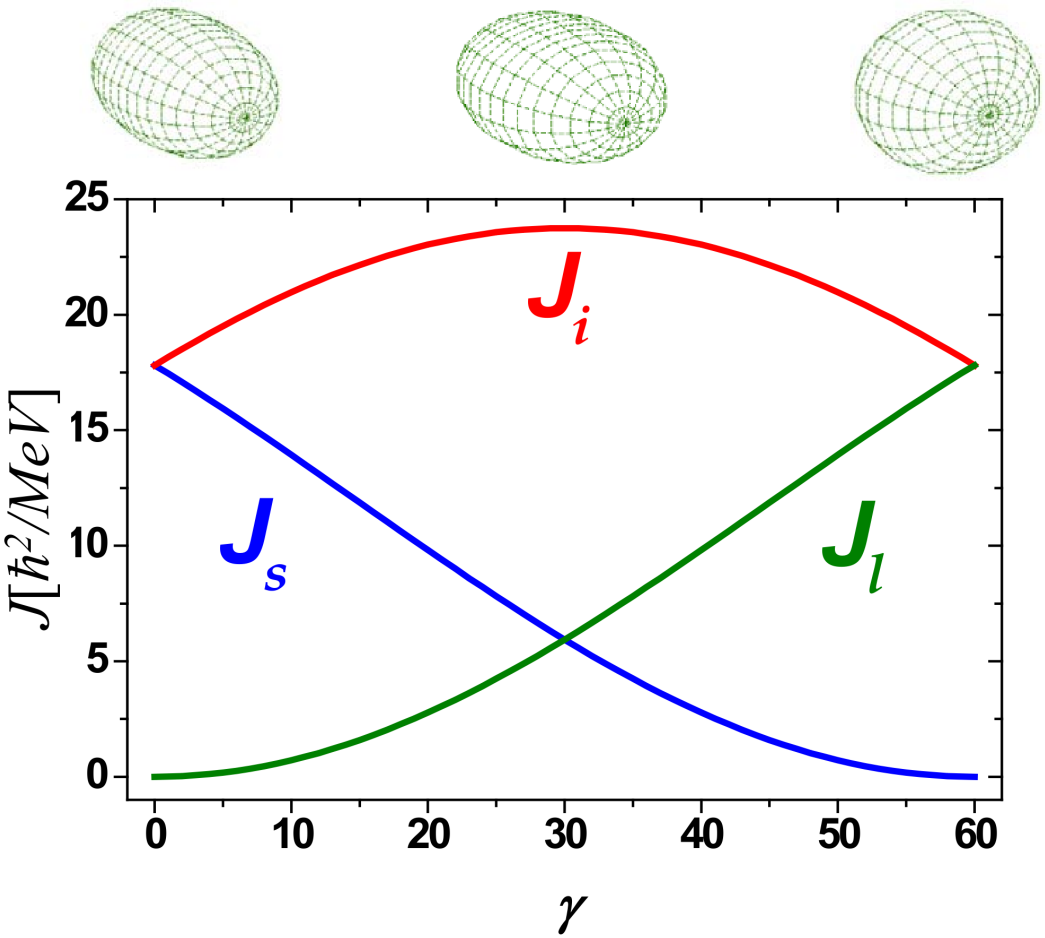
Irrotational flow moments of inertia



A. Bohr and B Mottelson, *Nuclear Structure* vol. II

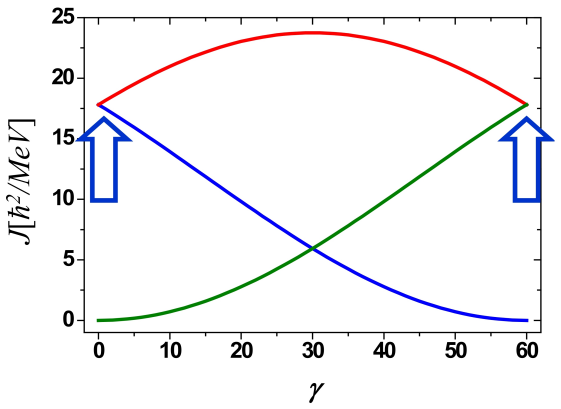
Irrotational flow moments of inertia

$$
\begin{aligned}
J_k &= 4B\beta^2 \sin^2(\gamma - k \cdot 120^\circ) \\
&= J_0 \sin^2(\gamma - k \cdot 120^\circ) \\
k &= 1, 2, 3 \\
&= \text{long}, \\
&\quad \text{short}, \\
&\quad \text{intermediate}
\end{aligned}
$$



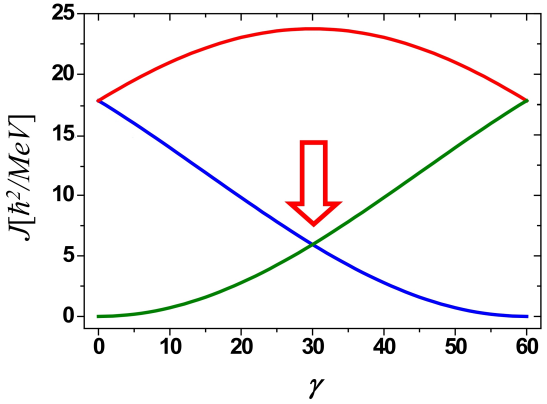
A. Bohr and B Mottelson, *Nuclear Structure* vol. II

Axial rotation of axial body



$$
\begin{aligned}
J_1 &= J_2 = \frac{3}{4} J_0, \quad J_3 = 0 \\
J_3 &= 0 \Rightarrow R_3 = 0 \Rightarrow R^2 = R_1^2 + R_2^2 \Rightarrow K = 0 \\
E_R &= \frac{2\hbar^2}{3J_0} R(R+1) \ \text{ for even } R
\end{aligned}
$$

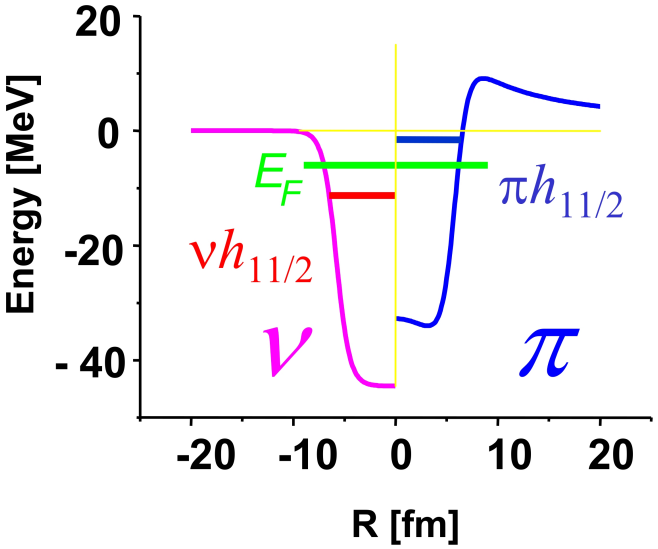
Axial rotation of triaxial body

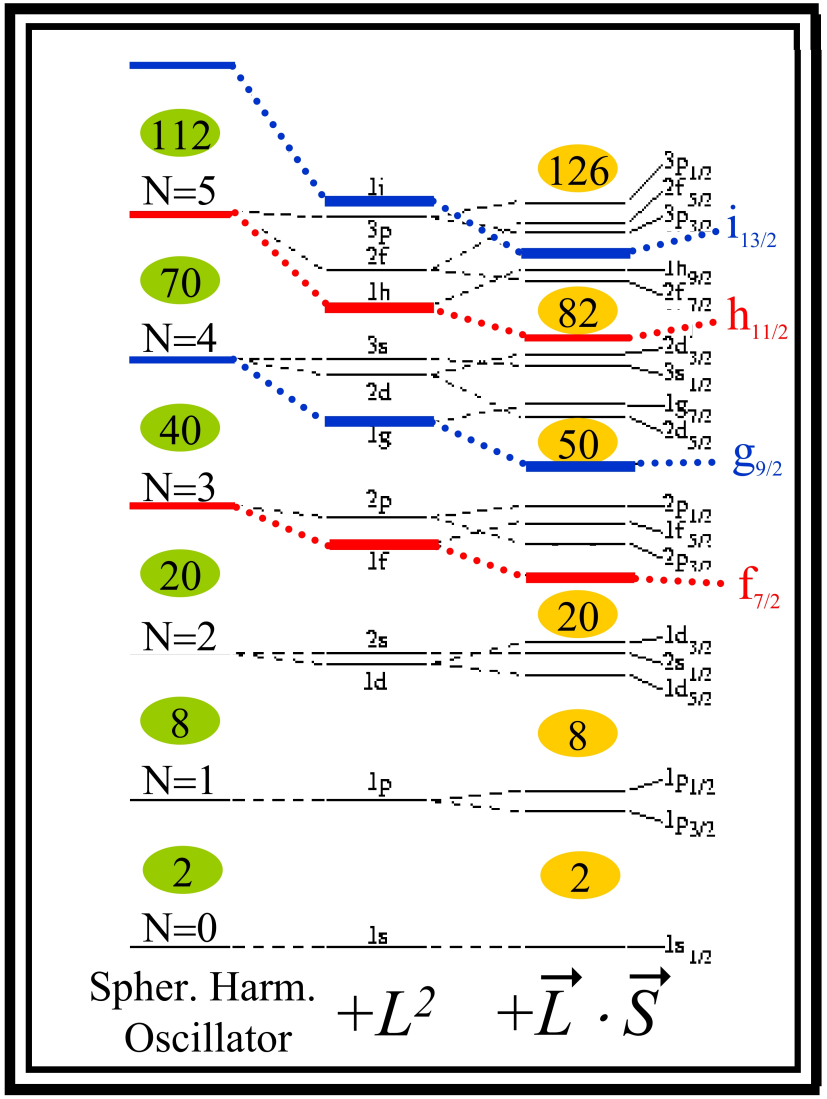


$$
\begin{aligned}
J_1 &= J_2 = \frac{1}{4}J_0, \quad J_3 = J_0 \\
H_R &= \sum_{k=1}^{3} \frac{R_k^2\hbar^2}{2J_k} = \frac{2\hbar^2}{J_0}((R_1^2 + R_2^2) + \frac{1}{4}R_3^2) = \\
& \frac{2\hbar^2}{J_0}((R^2 - R_3^2) + \frac{1}{4}R_3^2) = \frac{2\hbar^2}{J_0}(R^2 - \frac{3}{4}R_3^2) \\
E_{RK} &= \frac{2\hbar^2}{J_0}\left(R(R+1) - \frac{3}{4}K^2\right)
\end{aligned}
$$

Unique-parity orbitals in the nuclear Shell Model

$$H_{SM} = V(r) + V_{LS}(r)\,\vec{L}\cdot\vec{S}$$





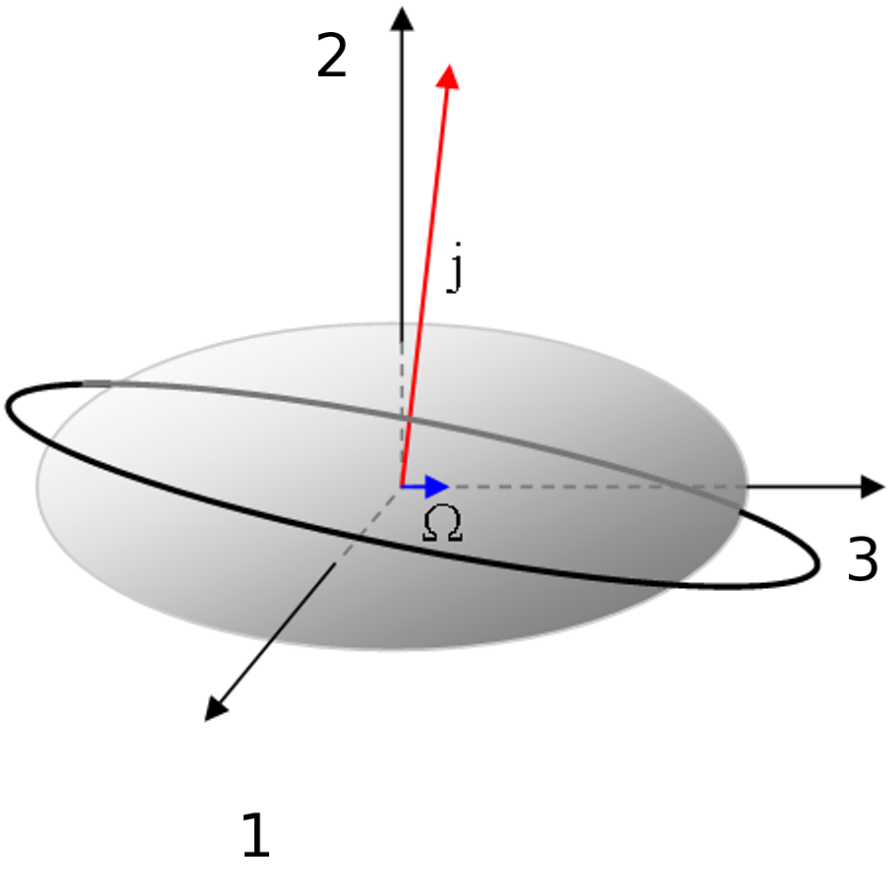
Deformed (Nilsson) Shell Model

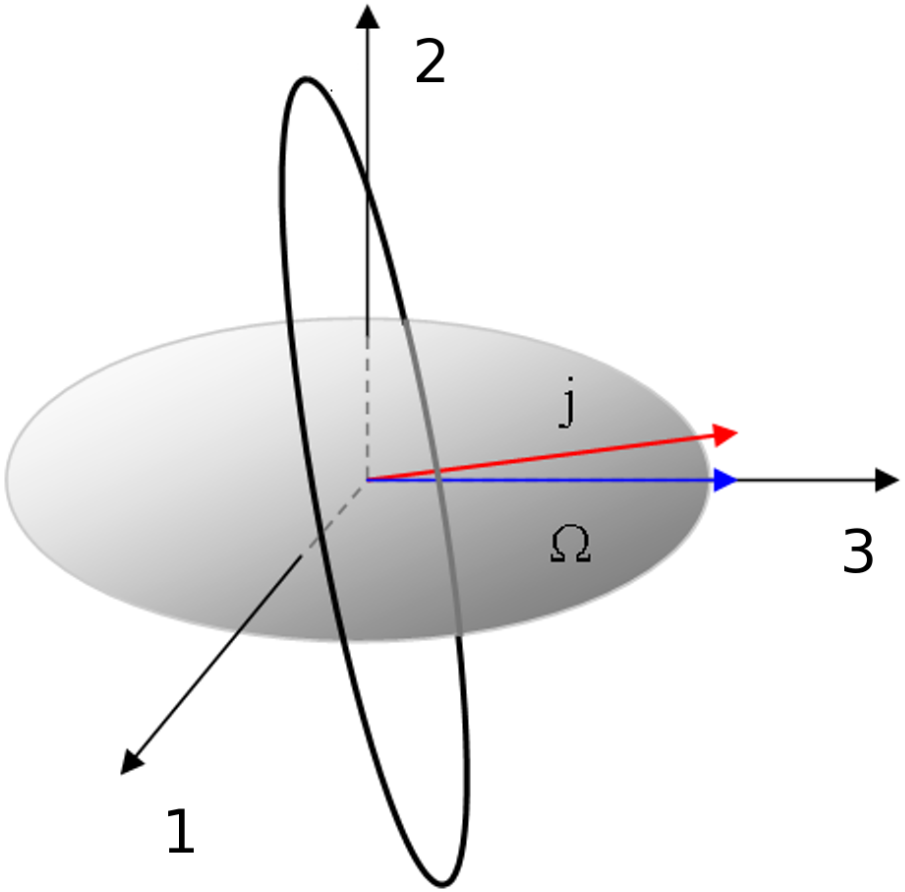
$$
\begin{aligned}
H_{sp} &= H_s + H_\beta \\
H_\beta(\beta, \gamma, r, \theta, \phi) &= -\kappa(r)\beta \left[\cos\gamma\, Y_{2,0}(\theta, \phi)\right. \\
&+ \left.\frac{1}{\sqrt{2}} \sin\gamma \left(Y_{2,2}(\theta, \phi) + Y_{2,-2}(\theta, \phi)\right)\right]
\end{aligned}
$$

Single-j approximation in deformed Shell Model

$$
\begin{aligned}
j_0 &= j_3, \\
j_{\pm 1} &= \mp \frac{1}{\sqrt{2}} \left(j_1 \pm i j_2 \right) \\
\left\langle j, \Omega \middle| Y_{2,0} \middle| j, \Omega' \right\rangle &= -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \frac{\left\langle j, \Omega \middle| 3 j_0^2 - j^2 \middle| j, \Omega' \right\rangle}{j(j+1)} \\
\left\langle j, \Omega \middle| Y_{2,\pm 2} \middle| j, \Omega' \right\rangle &= -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \sqrt{3} \frac{\left\langle j, \Omega \middle| j_{\pm 1}^2 \middle| j, \Omega' \right\rangle}{j(j+1)}
\end{aligned}
$$

Spin alignment through nuclear shape deformation





Single-j approximation in deformed Shell Model

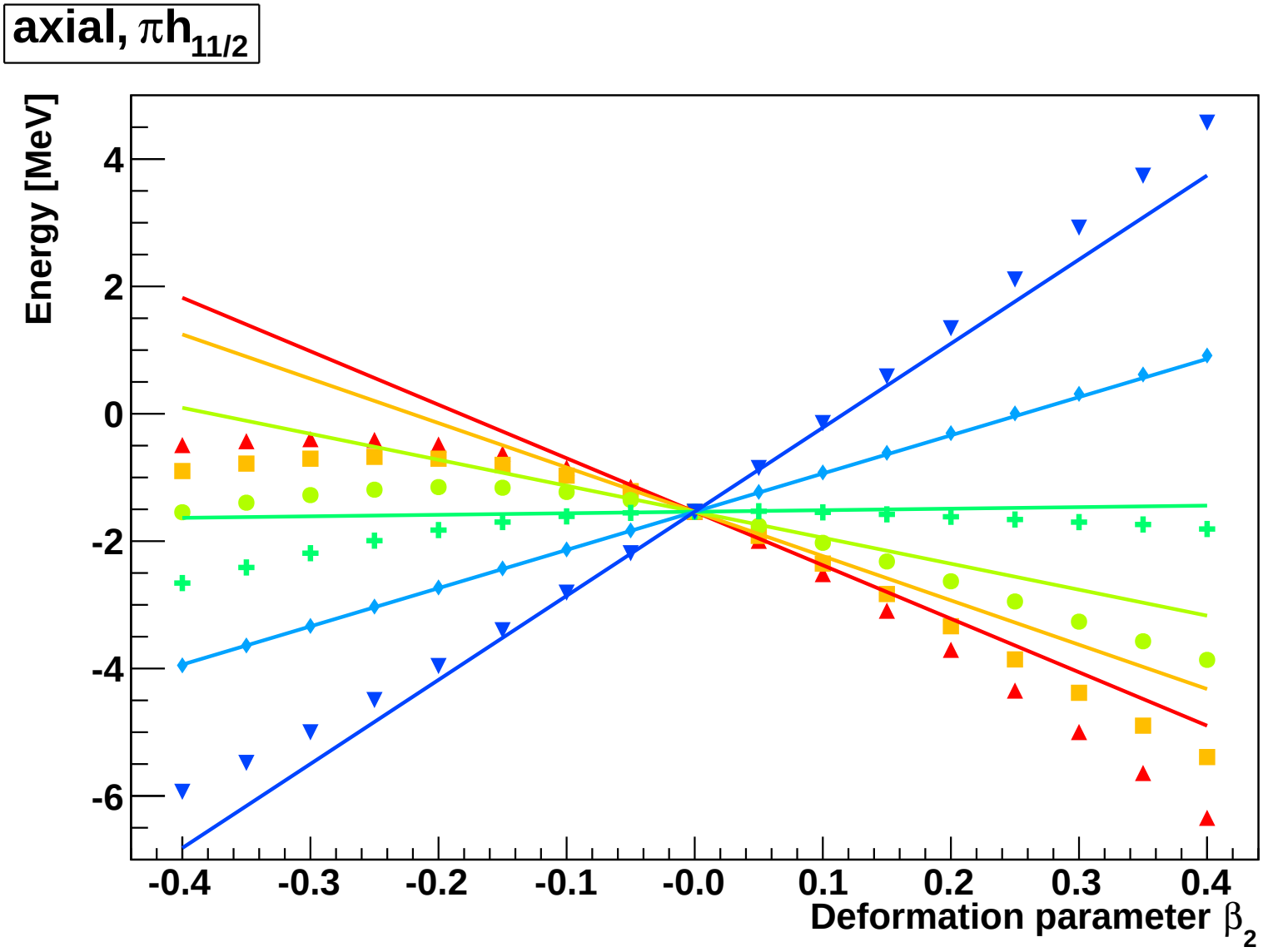
Axial deformation:

$$\begin{aligned} H_{sp} &= H_s + H_\beta = \\ &= H_s + \chi\beta\left(3j_0^2 - j^2\right) \end{aligned}$$

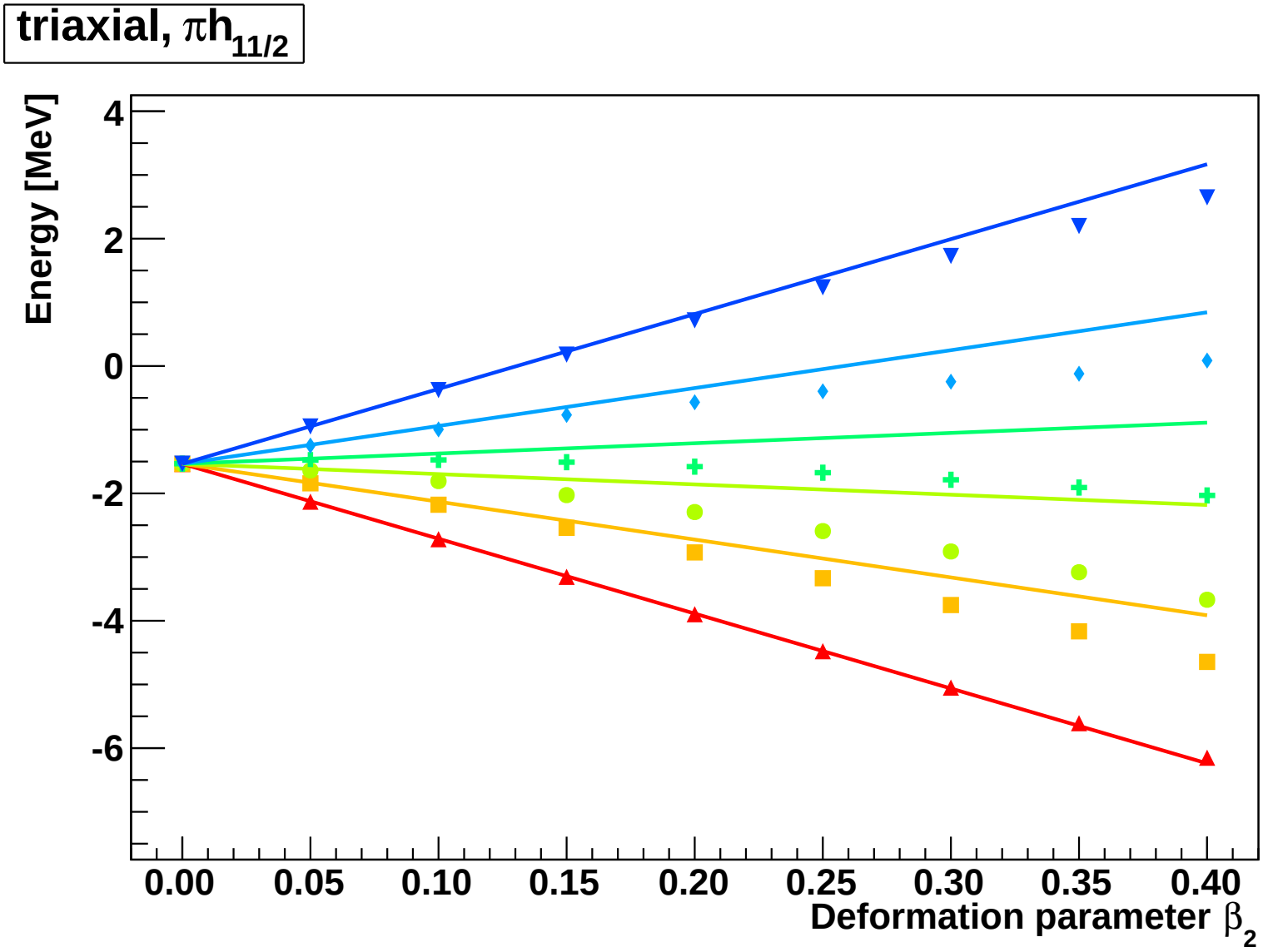
Triaxial deformation:

$$\begin{aligned} H_{sp} &= H_s + H_\beta = \\ &= H_s + \chi\beta\sqrt{3}\left(j_1^2 - j_2^2\right) \end{aligned}$$

Single-j approximation for $\pi h_{11/2}$ in axial potential



Single-j approximation for $\pi h_{11/2}$ in triaxial potential



Single-j approximation for $\pi h_{11/2}$ in triaxial potential

State	$E_q/\chi\beta$	$\langle j_1 \rangle$	$\langle j_1^2 \rangle$	$\langle j_2 \rangle$	$\langle j_2^2 \rangle$	$\langle j_3 \rangle$	$\langle j_3^2 \rangle$
$\lvert\Psi_{12}\rangle$	28.02	5.46	29.88	0.00	1.85	0.00	4.02
$\lvert\Psi_{11}\rangle$	28.02	-5.46	29.88	0.00	1.85	0.00	4.02
$\lvert\Psi_{10}\rangle$	14.17	4.32	19.04	0.00	4.87	0.00	11.83
$\lvert\Psi_{9}\rangle$	14.17	-4.32	19.04	0.00	4.87	0.00	11.83
$\lvert\Psi_{8}\rangle$	3.85	2.77	9.85	0.00	6.00	0.00	19.90
$\lvert\Psi_{7}\rangle$	3.85	-2.77	9.85	0.00	6.00	0.00	19.90
$\lvert\Psi_{6}\rangle$	-3.85	0.00	6.00	2.77	9.85	0.00	19.90
$\lvert\Psi_{5}\rangle$	-3.85	0.00	6.00	-2.77	9.85	0.00	19.90
$\lvert\Psi_{4}\rangle$	-14.17	0.00	4.87	4.32	19.04	0.00	11.83
$\lvert\Psi_{3}\rangle$	-14.17	0.00	4.87	-4.32	19.04	0.00	11.83
$\lvert\Psi_{2}\rangle$	-28.02	0.00	1.85	5.46	29.88	0.00	4.02
$\lvert\Psi_{1}\rangle$	-28.02	0.00	1.85	-5.46	29.88	0.00	4.02

Particle-hole-triaxial-rotor model

$$
\begin{aligned}
H &= H_R + H_\pi - H_\nu = H_d + H_p \\
H_d &= \frac{2\hbar^2}{J_0}\left(I^2 + j_\pi^2 + j_\nu^2 - \frac{3}{4}I_0^2\right) - \sqrt{3}\chi\beta\left(j_{\pi 2}^2 + j_{\nu 1}^2\right) \\
H_p &= H_{pIj\pi} + H_{pIj_\nu} + H_{pj_\pi j_\nu} + H_{p2} \\
H_{pIj\pi} &= \frac{2\hbar^2}{J_0}\left(2I_{+1}j_{\pi -1} + 2I_{-1}j_{\pi +1} - \frac{1}{2}I_0 j_{\pi 0}\right) \\
H_{pIj\nu} &= \frac{2\hbar^2}{J_0}\left(2I_{+1}j_{\nu -1} + 2I_{-1}j_{\nu +1} - \frac{1}{2}I_0 j_{\nu 0}\right) \\
H_{pj\pi j_\nu} &= -\frac{2\hbar^2}{J_0}\left(2j_{\pi +1}j_{\nu -1} + 2j_{\pi -1}j_{\nu +1} - \frac{1}{2}j_{\pi 0}j_{\nu 0}\right) - \frac{3\hbar^2}{2J_0}\left(j_{\pi 0}^2 + j_{\nu 0}^2\right) \\
H_{p2} &= \sqrt{3}\chi\beta\left(j_{\pi 1}^2 + j_{\nu 2}^2\right)
\end{aligned}
$$