Chiral basis for particle-rotor model for odd-odd triaxial nuclei

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Chirality in atomic nuclei

Early experimental searches
Orthogonal coupling of three vectors
Molecular (geometric) chirality
Chirality unique to atomic nuclei

Atomic nuclei are the only systems which can provide two single-particle angular momenta components needed for angular-momentum chirality.
Doublet bands in triaxial regions
Doublet bands in triaxial regions

Colour denotes energy gain resulting from activating the triaxial degrees of freedom in the Möller-Nix mass model.
Nuclear chirality

Signature of irrotational flow

\[ J_{k}^{irr.} = J_0 \frac{4}{3} \sin^2 (\gamma - \frac{2}{3} \pi k) \]

\[ J_{k}^{rig.} = J_0 (1 + \sqrt{\frac{5}{4\pi}} \beta \cos(\gamma - \frac{2}{3} \pi k)) \]

irrotational flow

rigid body

Energy [MeV]

Spin [\hbar]

Energy [MeV]

Spin [\hbar]
Is there a smoking gun?
Circumstantial evidence

Is there a smoking gun?

- For cases with the smallest energy separation between doublet states, $^{134}$Pr and $^{104}$Rh both with $\Delta E < 50$ keV, inconsistent behaviour of the electromagnetic properties is a problem.

- In other cases, if the transition rates are all right, the degeneracy of levels is not that great.

- There is no single case which would show in a satisfactory way all properties expected of nuclear chirality.

- Various scenarios are used to explain this fact, most notably, tunnelling through the barrier enabling chiral vibrations of the total angular momentum.
Back to the basics

- Despite $\sim 20$ years of research chirality is extracted from expectation values of orientation operators, rather than being a starting point in construction of nuclear models.

- In that sense, chirality has been perceived as an approximate symmetry attained only in a limited range of angular momentum.

- However, using the particle-hole-coupling model for triaxial odd-odd nuclei it is possible to construct a basis which contains right-handed, left-handed, and planar states of angular momentum coupling.

- If this basis is used, the chirality is explicit rather than extracted feature as in any other models with non-chiral basis.
\[ |RM\rangle = \sum_{K=-R}^{K=R} D^R_{MK}(\omega) |RK\rangle \]

\( R \) angular momentum

\( M \) z-projection of \( R \)

\( K \) 3-projection of \( R \)

\( \omega = (\phi, \theta, \psi) \) Euler angles
Standard particle-hole-rotor basis
Chiral particle-hole-rotor basis
Orthogonal coupling of three vectors
The chiral basis results from a $\pi/2$ rotation of standard single-particle proton/neutron states around the 1st (long) axis and a $\pi/2$ rotation of standard single-particle neutron/proton states around the 2nd (short) axis

$$|j_\kappa\rangle = \sum_\Omega d^j_{\kappa\Omega} \left(-\pi/2\right) |j\Omega\rangle$$

$$|j_\nu\rangle = \sum_\Omega (-i)^{\kappa-\Omega} d^j_{\nu\Omega} \left(-\pi/2\right) |j\Omega\rangle$$

$\kappa/\nu$ projection of $j_\nu/j_\pi$ on the long/short axis.

**Correlations built into the basis prior to diagonalization.**
The chiral basis

Axial rotation of triaxial body

\[ J_1 = J_2 = \frac{1}{4} J_0, \quad J_3 = J_0 \]

\[ H_R = \sum_{k=1}^{3} \frac{R_k^2 \hbar^2}{2 J_k} = \frac{2 \hbar^2}{J_0} \left( (R_1^2 + R_2^2) + \frac{1}{4} R_3^2 \right) = \]

\[ \frac{2 \hbar^2}{J_0} \left( (R^2 - R_3^2) + \frac{1}{4} R_3^2 \right) = \frac{2 \hbar^2}{J_0} \left( R^2 - \frac{3}{4} R_3^2 \right) \]

\[ E_{RK} = \frac{2 \hbar^2}{J_0} \left( R(R + 1) - \frac{3}{4} K^2 \right) \]
$D_2$ symmetry of the Hamiltonian

\[ \mathcal{R}_\kappa^2(\pi) = 1, \quad \kappa = 1, 2, 3 \]

\[ [R_k^2, \mathcal{R}_\kappa(\pi)] = 0, \quad k = 1, 2, 3, \quad \kappa = 1, 2, 3 \]

\[ [H_R, \mathcal{R}_\kappa(\pi)] = 0, \quad \kappa = 1, 2, 3 \]
The chiral basis

Single-$j$ approximation in the deformed mean field

\[ j_0 = j_3, \]

\[ j_{\pm 1} = \pm \frac{1}{\sqrt{2}} (j_1 \pm ij_2) \]

\[ \langle j, \Omega | Y_{2,0} | j, \Omega' \rangle = -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \frac{\langle j, \Omega | 3j_0^2 - j^2 | j, \Omega' \rangle}{j(j+1)} \]

\[ \langle j, \Omega | Y_{2,\pm 2} | j, \Omega' \rangle = -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \sqrt{3} \frac{\langle j, \Omega | j_{\pm 1}^2 | j, \Omega' \rangle}{j(j+1)} \]
Particle-hole-triaxial-rotor model chiral wave functions

$$| IMKj_\pi \kappa_\pi j_\nu \kappa_\nu \rangle = \frac{1}{2} \sqrt{\frac{2I + 1}{8\pi^2}} \left( D_{MK}^I(\omega) | j_\pi \kappa_\pi \rangle | j_\nu \kappa_\nu \rangle ight)$$

$$+ (-1)^{I + j_\pi + \kappa_\nu} D_{MK}^I(\omega) | j_\pi \bar{\kappa}_\pi \rangle | j_\nu \kappa_\nu \rangle$$

$$+ (-1)^{I - j_\nu + \kappa - \kappa_\pi - \kappa_\nu} D_{MK}^I(\omega) | j_\pi \kappa_\pi \rangle | j_\nu \bar{\kappa}_\nu \rangle$$

$$+ (-1)^{j_\pi + j_\nu + \kappa - \kappa_\pi} D_{MK}^I(\omega) | j_\pi \bar{\kappa}_\pi \rangle | j_\nu \bar{\kappa}_\nu \rangle$$
Particle-hole-triaxial-rotor model chiral wave functions

\[
|IMKj_\pi \kappa_\pi j_\nu \kappa_\nu\rangle = \frac{1}{2} \sqrt{\frac{2l + 1}{8\pi^2}} \left( D^l_{MK}(\omega) |j_\pi \kappa_\pi\rangle |j_\nu \kappa_\nu\rangle \right.
+ (-1)^{l+j_\pi+\kappa_\nu} D^l_{MK}(\omega) |j_\pi \kappa_\pi\rangle |j_\nu \kappa_\nu\rangle
+ (-1)^{l-j_\nu+K-\kappa_\pi-\kappa_\nu} D^l_{MK}(\omega) |j_\pi \kappa_\pi\rangle |j_\nu \kappa_\nu\rangle
+ (-1)^{j_\pi+j_\nu+K-\kappa_\pi} D^l_{MK}(\omega) |j_\pi \kappa_\pi\rangle |j_\nu \kappa_\nu\rangle \left. \right)
\]

\[K = 0 \text{ or } \kappa = \pm \frac{1}{2} \text{ or } \kappa = \frac{1}{2} : \text{ planar states}\]

\[K > 0, \kappa \in \left[\frac{3}{2}, j\right], \kappa \in \left[\frac{3}{2}, j\right] : \text{ right-handed states}\]

\[K > 0, \kappa \in \left[-j, -\frac{3}{2}\right], \kappa \in \left[\frac{3}{2}, j\right] : \text{ left-handed states}\]
Planar states

\[ O |I P\rangle = TR_y(\pi) |I P\rangle = |I P\rangle . \]

\[ TR_y(\pi) \begin{array}{c}
\end{array} = T \begin{array}{c}
\end{array} = \begin{array}{c}
\end{array} \]
Chiral states

\[ O |I\,R\rangle = T \, R_y(\pi) |I\,R\rangle = |I\,L\rangle , \]
\[ O |I\,L\rangle = T \, R_y(\pi) |I\,L\rangle = |I\,R\rangle . \]

\[ T \, R_y(\pi) | \begin{array}{c}
\text{Top right arrow}
\end{array} \rangle = T | \begin{array}{c}
\text{Bottom left arrow}
\end{array} \rangle = | \begin{array}{c}
\text{Top left arrow}
\end{array} \rangle \]
The chiral basis

The Hamiltonian

standard

chiral
The Hamiltonian

standard

chiral
Eigen states of the Hamiltonian

Energy [MeV]

full  right-handed  left-handed  planar

-1  0  1  2  3  4
The chiral basis

\( \pi h_{11/2} \nu h^{-1}_{11/2} \) particle-hole-triaxial-rotor model

$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model
The chiral basis

$\pi h_{11/2} \nu h_{11/2}^{-1}$ particle-hole-triaxial-rotor model

Probability

$I=8$

$I=9$

$I=14$

$I=15$

$-9/2$  $-1/2$  $7/2$

$-9/2$  $-1/2$  $7/2$
The chiral basis

\[ \pi h^{11/2} \nu h^{-1}_{11/2} \] particle-hole-triaxial-rotor model

![Graphs showing probability distribution for different angular momenta (I=8, I=9, I=14, I=15)]
Conclusions

- A new basis is proposed for the triaxial particle-rotor model.
- It is applicable to particle/hole coupling in odd- and odd-odd nuclei.
- High-\(j\) particle and high-\(j\) hole coupling to the triaxial core results in basis states which are left-handed, right-handed, and planar.
- Chiral basis diagonalize a significant fraction of the model Hamiltonian providing efficient expansion for wave functions of final states.
- Calculations confirm doubling of states arising from orthogonal coupling of angular momentum vectors.
- The wave function expanded in the left-handed, right-handed, and planar states will be used to identify and investigate observables sensitive to chirality in angular momentum coupling.
Is there a smoking gun?

The Truth Is Out There.
Standard particle-rotor basis
New ("chiral") particle-rotor basis
Wave function in the standard and in the chiral basis

$I = 21/2$

standard

chiral
Conclusions

Quadrupole deformation

spherical  axial  triaxial
Hamiltonian for a rotor

\[ H_R = \hbar^2 \left( \frac{R_1^2}{J_1} + \frac{R_2^2}{J_2} + \frac{R_3^2}{J_3} \right) \]

\( R_i \) for \( i = 1, 2, 3 \) are rotor’s angular momentum components

\( J_i \) for \( i = 1, 2, 3 \) are rotor’s principle-axes moments of inertia
Irrotational flow moments of inertia

A. Bohr and B Mottelson, *Nuclear Structure* vol. II

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Irrotational flow moments of inertia

\[ J_k = 4B\beta^2 \sin^2(\gamma - k \cdot 120^\circ) \]

\[ = J_0 \sin^2(\gamma - k \cdot 120^\circ) \]

\[ k = 1, 2, 3 \]

= long, short, intermediate

A. Bohr and B. Mottelson, *Nuclear Structure* vol. II
Axial rotation of axial body

\[ J_1 = J_2 = \frac{3}{4} J_0, \quad J_3 = 0 \]

\[ J_3 = 0 \Rightarrow R_3 = 0 \Rightarrow R^2 = R_1^2 + R_2^2 \Rightarrow K = 0 \]

\[ E_R = \frac{2\hbar^2}{3J_0} R(R + 1) \text{ for even } R \]
Axial rotation of triaxial body

\[ J_1 = J_2 = \frac{1}{4} J_0, \quad J_3 = J_0 \]

\[ H_R = \sum_{k=1}^{3} \frac{R_k^2 \hbar^2}{2 J_k} = \frac{2 \hbar^2}{J_0} \left( \left( R_1^2 + R_2^2 \right) + \frac{1}{4} R_3^2 \right) = \frac{2 \hbar^2}{J_0} \left( \left( R^2 - R_3^2 \right) + \frac{1}{4} R_3^2 \right) = \frac{2 \hbar^2}{J_0} \left( R^2 - \frac{3}{4} R_3^2 \right) \]

\[ E_{RK} = \frac{2 \hbar^2}{J_0} \left( R(R + 1) - \frac{3}{4} K^2 \right) \]
Unique-parity orbitals in the nuclear Shell Model

\[ H_{SM} = V(r) + V_{LS}(r) \hat{L} \cdot \hat{S} \]

Energy [MeV]

\[ E_F \]

\[ \nu h_{11/2} \]

\[ \pi h_{11/2} \]

\[ R [fm] \]

Spher. Harm. Oscillator

\[ + L^2 \]

\[ + \hat{L} \cdot \hat{S} \]
\[ H_{sp} = H_s + H_\beta \]

\[ H_\beta(\beta, \gamma, r, \theta, \phi) = -\kappa(r)\beta [\cos \gamma Y_{2,0}(\theta, \phi) \]

\[ + \frac{1}{\sqrt{2}} \sin \gamma (Y_{2,2}(\theta, \phi) + Y_{2,-2}(\theta, \phi)) \]
Single-\(j\) approximation in deformed Shell Model

\[\begin{align*}
j_0 & = j_3, \\
j_{\pm 1} & = \pm \frac{1}{\sqrt{2}} (j_1 \pm ij_2) \\
\langle j, \Omega | Y_{2,0} | j, \Omega' \rangle & = -\frac{1}{4} \sqrt{\frac{5}{4\pi}} j(j + 1) \frac{\langle j, \Omega | 3j_0^2 - j^2 | j, \Omega' \rangle}{j_0(j + 1)} \\
\langle j, \Omega | Y_{2,\pm 2} | j, \Omega' \rangle & = -\frac{1}{4} \sqrt{\frac{5}{4\pi}} \sqrt{3} \frac{\langle j, \Omega | j_{\pm 1}^2 | j, \Omega' \rangle}{j(j + 1)}
\end{align*}\]
Spin alignment through nuclear shape deformation
Single-$j$ approximation in deformed Shell Model

Axial deformation:

\[ H_{sp} = H_s + H_\beta = \]
\[ = H_s + \chi \beta (3j_0^2 - j^2) \]

Triaxial deformation:

\[ H_{sp} = H_s + H_\beta = \]
\[ = H_s + \chi \beta \sqrt{3} (j_1^2 - j_2^2) \]
Single-$j$ approximation for $\pi h_{11/2}$ in axial potential

![Graph showing the relationship between energy (in MeV) and deformation parameter $\beta_2$ for axial, $\pi h_{11/2}$](image-url)
Single-\(j\) approximation for \(\pi h_{11/2}\) in triaxial potential
### Single-$j$ approximation for $\pi \hbar_{11/2}$ in triaxial potential

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<th>$\langle j_1 \rangle$</th>
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Particle-hole-triaxial-rotor model

\[ H = H_R + H_\pi - H_\nu = H_d + H_p \]
\[ H_d = \frac{2\hbar^2}{J_0} \left( I^2 + j_\pi^2 + j_\nu^2 - \frac{3}{4} I_0^2 \right) - \sqrt{3} \chi \beta \left( j_{\pi2}^2 + j_{\nu1}^2 \right) \]
\[ H_p = H_{plj_\pi} + H_{plj_\nu} + H_{pj_\pi j_\nu} + H_{p2} \]
\[ H_{plj_\pi} = \frac{2\hbar^2}{J_0} \left( 2I_+ j_{\pi-1} + 2I_- j_{\pi+1} - \frac{1}{2} I_0 j_{\pi0} \right) \]
\[ H_{plj_\nu} = \frac{2\hbar^2}{J_0} \left( 2I_+ j_{\nu-1} + 2I_- j_{\nu+1} - \frac{1}{2} I_0 j_{\nu0} \right) \]
\[ H_{pj_\pi j_\nu} = -\frac{2\hbar^2}{J_0} \left( 2j_{\pi+1} j_{\nu-1} + 2j_{\pi-1} j_{\nu+1} - \frac{1}{2} j_{\pi0} j_{\nu0} \right) - \frac{3\hbar^2}{2J_0} \left( j_{\pi0}^2 + j_{\nu0}^2 \right) \]
\[ H_{p2} = \sqrt{3} \chi \beta \left( j_{\pi1}^2 + j_{\nu2}^2 \right) \]