Enhanced Collectivity of Gamma Vibration in Neutron-rich Dy isotopes with $N=108-110$

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Gamma vibration

- Systematically appears in the even-even deformed nuclei
- Low-frequency quadrupole vibration along the γ direction
- Soft mode of the triaxial deformation

How about in neutron-rich deformed nuclei?

universality of emergence of the collective mode across the nuclear chart
Collectivity around $^{170}_{66}$Dy$_{104}$

Middle of the major shells between $^{132}$Sn and $^{208}$Pb

✓ Effect of neutron excess on the occurrence of $\gamma$-vibration
✓ Shell structure in neutron-rich medium-heavy nuclei

Decay spectroscopy at RIKEN RIBF: EURICA

H. Watanabe et al., PLB760(2016)641

$^{172}$Dy$_{106}$

Long-lived isomeric state

Ground-state rotational band

Possible gamma band

Sudden decrease in the excitation energy at $N=106$
Nuclear DFT for collective vibration w/ Skyrme + pairing energy-density functional

\[ \mathcal{E}[\rho, \tilde{\rho}](r) \]

Hartree-Fock-Bogoliubov (HFB) like equation

\[
\begin{pmatrix}
\tilde{h}^q(r\sigma) - \lambda^q \\
\tilde{h}^q(r\sigma) - (h(r\sigma) - \lambda^q)
\end{pmatrix}
\begin{pmatrix}
\varphi_1^q(\sigma) \\
\varphi_2^q(\sigma)
\end{pmatrix}
= E^q
\begin{pmatrix}
\varphi_1^q(\sigma) \\
\varphi_2^q(\sigma)
\end{pmatrix}
\]

"s.p." hamiltonian and pair potential:

\[
h^q = \frac{\delta \mathcal{E}}{\delta \rho^q}, \quad \tilde{h}^q = \frac{\delta \mathcal{E}}{\delta \tilde{\rho}^q}
\]

response to the weak external field

\[ \hat{F} : \quad \nu^{\text{ext}}(r)e^{-i\omega t} \]

QRPA:

\[
\delta \rho_i(r) = \int dr' \chi_{ij}^0(r, r') \left[ \frac{\delta^2 \mathcal{E}}{\delta \rho_j \delta \rho_k} \delta \rho_k(r') + \nu_j^{\text{ext}}(r') \right]
\]

transition matrix elements:

\[
\langle \Psi_\lambda | \hat{F} | \Psi_0 \rangle = \int dr \delta \rho(r; \omega_\lambda) \nu^{\text{ext}}(r)
\]

J. Dobaczewski et al., NPA422(1984)103
Single-particle energies in $^{172}$Dy

Proton single-particle energy (MeV)

- $g_{7/2}$
- $d_{5/2}$
- $h_{11/2}$
- $d_{3/2}$
- $[411]1/2$
- $[411]3/2$
- $[413]5/2$

Deformation parameter $\beta$

Neutron single-particle energy (MeV)

- $h_{11/2}$
- $f_{7/2}$
- $h_{9/2}$
- $i_{13/2}$
- $p_{3/2}$
- $p_{1/2}$

$\Delta N = 0$ or 2, $\Delta n_3 = 0$, $\Delta \Lambda = \Delta \Omega = 2$

Deformation parameter $\beta$

SkM*
γ-vibration in the neutron-rich Dy isotopes

![Graph showing QRPA frequencies for different neutron numbers (N) with markers for SkM*, SLy4, NNDC, and RIBF. The graph indicates a softening of γ-vibration with increasing neutron number.]
Microscopic structure of the $\gamma$-vibration: isotopic dependence

$@N=102$

$\Delta N = 0$ or 2, $\Delta n_3 = 0$, $\Delta \Lambda = \Delta \Omega = 2$

\begin{align*}
\langle \Psi_{\lambda} | \hat{F} | \Psi_0 \rangle &= \int \delta \rho(r; \omega_{\lambda}) v_{\text{ext}}(r) \, dr \\
\delta \rho_i(r) &= \int \chi_{ij}(r, r') \left[ \delta^2 E \delta \rho_j \delta \rho_k(r') + v_{\text{ext}}(r) \right] e^{-i \omega t} h_q = \delta E \delta \rho_q, \tilde{h}_q = \delta E \delta \tilde{\rho}_q(r_{\sigma}) - \lambda q \tilde{h}_q(r_{\sigma}) - (h(r_{\sigma}) - \lambda q) \phi_{q 1, \alpha}(r_{\sigma}) \phi_{q 2, \alpha}(r_{\sigma}) = E_{\alpha}(\phi_{q 1, \alpha}(r_{\sigma}) \phi_{q 2, \alpha}(r_{\sigma}))
\end{align*}
Microscopic structure of the γ-vibration: isotopic dependence

@N=104

\[ \Delta N = 0 \text{ or } 2, \Delta n_3 = 0, \Delta \Lambda = \Delta \Omega = 2 \]

![Graph](image-url)

**QRPA frequency (MeV)**

- SkM*
- NNDC

**Neutron single-particle energy (MeV)**

- Dy
- [510]1/2
- [512]5/2

**Deformation parameterer β**

- N = 104
- h_{11/2}
- h_{7/2}
- f_{7/2}
Microscopic structure of the $\gamma$-vibration: isotopic dependence

@N=106

$\Delta N = 0$ or 2, $\Delta n_3 = 0$, $\Delta \Lambda = \Delta \Omega = 2$

$\begin{align*}
\langle \Psi_{\lambda} | \hat{F} | \Psi_{0} \rangle &= \int \delta \rho_i (r; \omega_{\lambda}) v_{\text{ext}} (r) \\
\delta \rho_i (r) &= \int dr' \chi_{ij0} (r, r') \\

\delta^2 E \delta \rho_j \delta \rho_k \delta \rho_k (r') + v_{\text{ext}} j (r') \\
v_{\text{ext}} (r) e^{-i \omega t} h_q &= \delta E \delta \rho q, \\
\tilde{h}_q &= \delta E \delta \tilde{\rho} q \\
h_q (r_\sigma) - \lambda q \tilde{h}_q (r_\sigma) - \tilde{h}_q (r_\sigma) - (h (r_\sigma) - \lambda q) = (\phi q_1, \alpha (r_\sigma) \phi q_2, \alpha (r_\sigma)) = E_\alpha (\phi q_1, \alpha (r_\sigma) \phi q_2, \alpha (r_\sigma))
\end{align*}$
Microscopic structure of the $\gamma$-vibration: isotopic dependence

$@N=108$

$\Delta N = 0 \text{ or } 2, \Delta n_3 = 0, \Delta \Lambda = \Delta \Omega = 2$

![Graph showing the QRPA frequency of SkM* and NNDC for Dy as a function of neutron single-particle energy](image)

- $f_{5/2}$
- $p_{1/2}$
- $p_{3/2}$
- $h_{9/2}$
- $h_{11/2}$
- $[512]3/2$
- $[512]5/2$
- $[514]7/2$
- $[510]1/2$

Deformation parameter $\beta$
Strong collectivity of the $\gamma$-vibration around $N=108-110$

2qp matrix elements constructing the $\gamma$-vib.

$$\langle i | \hat{F}^q_{\chi K} | 0 \rangle = \sum_{\alpha \beta} M_{\alpha \beta}^{q,i}$$
Summary

✓ A sudden decrease in the excitation energy of the $\gamma$-vibration at $N=106$ was observed at RIBF for neutron-rich Dy isotopes.

✓ The EDF-based QRPA calculations with the SkM* and SLy4 functionals reproduce well the measurement.

✓ The two-quasiparticle excitations of $\nu[512]3/2 \otimes \nu[510]1/2$, $\nu[510]1/2 \otimes \nu[512]5/2$, and $\nu[512]3/2 \otimes \nu[514]7/2$ play a major role in generating the collective $\gamma$-vibration.

✓ The strong collectivity at $N = 108-110$ is expected as the Fermi level of neutrons lies just among the orbitals that play an important role in generating the collectivity around $N = 106$. 

arXiv:1607.07111
The left panel shows the deformation parameter as a function of the number of neutrons ($N$) for different elements: Dy, Er, and Yb. The right panel displays the QRPA frequency (in MeV) also as a function of $N$ for $Z=66$, $Z=68$, and $Z=70$. Both panels use the SkM* interaction model.