A TRANSITION-CENTRIC APPROACH TO NUCLEAR LEVEL SCHEME DETERMINATION

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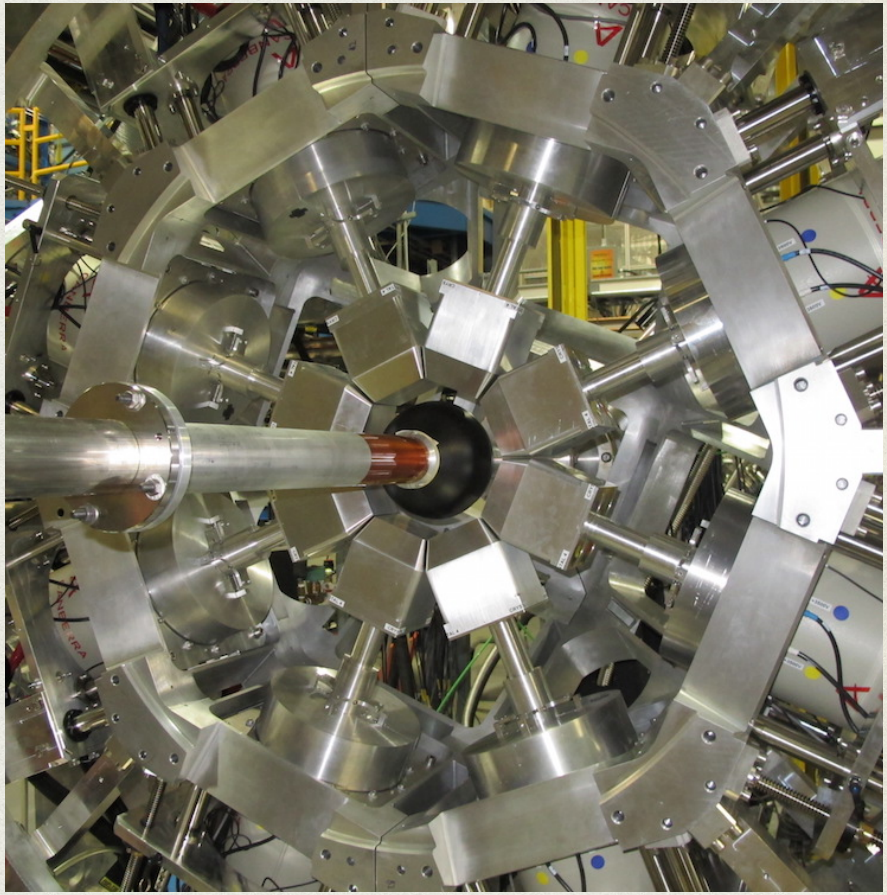
γ-RAY COINCIDENCE SPECTROSCOPY

γ-ray coincidence spectroscopy is a powerful, but time consuming method of nuclear level scheme determination

γ-ray coincidence spectroscopy measures the probability of detecting two γ-rays at the same time



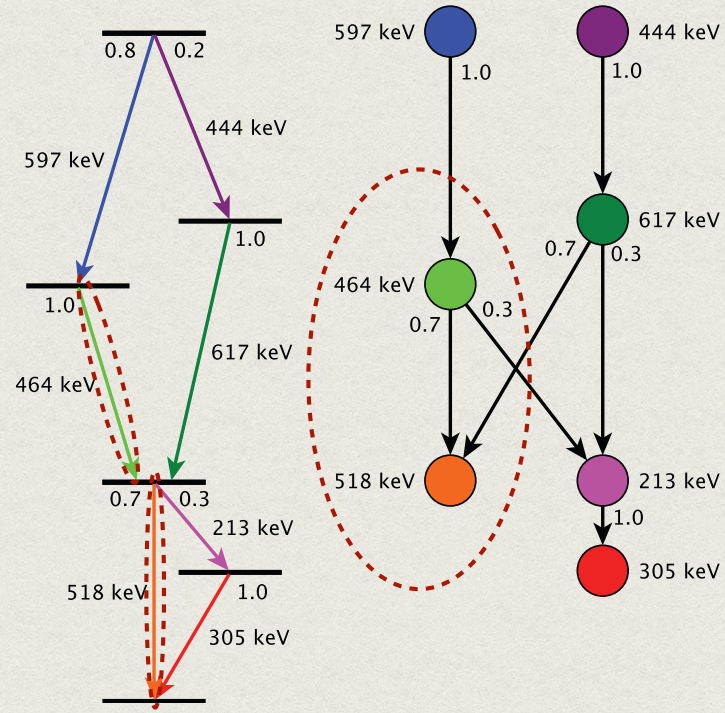
The GRIFFIN Spectrometer

TRANSITION-CENTRIC LEVEL SCHEMES

Level schemes can be represented in a transition-centric form that matches the form of the experimental data

Transitions are represented as nodes at their source excitation energy level

Connections between transitions (i.e. levels) are edges weighted by branching ratios



$$A = \left(\begin{array}{c|ccccccc} keV & 213 & 305 & 444 & 464 & 518 & 597 & 617 \\ \hline 213 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 305 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 444 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 464 & 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 518 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 597 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 617 & 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \end{array}\right)$$

ANALYTICAL FORMULA

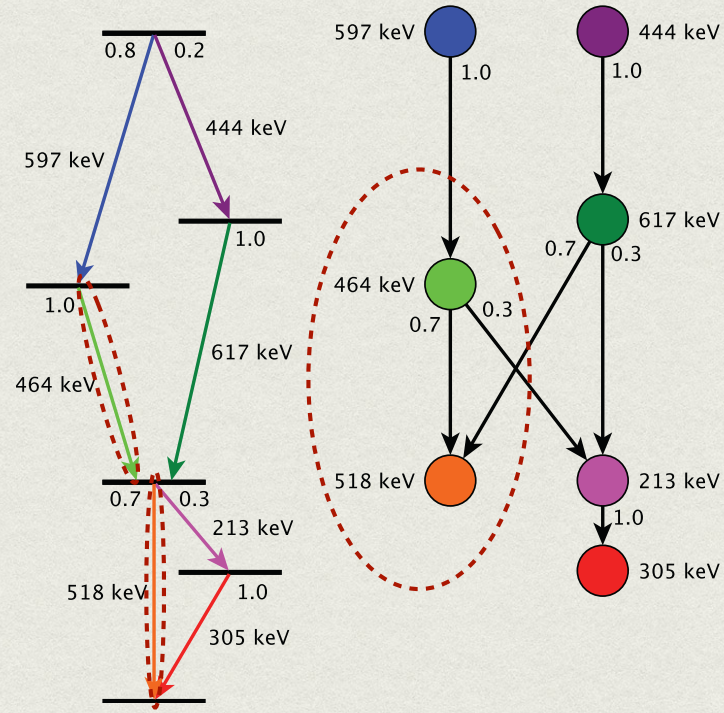
We can now derive an analytical formula that relates experimental data to level scheme structure

$$A = I - (S^{-1}DE^{-1} + I)^{-1}$$

$$C = D + D^T$$

$D =$ Directed Coincidence Data
$C =$ Coincidence Data
$S =$ Singles Data
$E =$ Efficiencies

$A =$ Adjacency Matrix (level scheme)



$$A = \left(\begin{array}{c|ccccccc} keV & 213 & 305 & 444 & 464 & 518 & 597 & 617 \\ \hline 213 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 305 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 444 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 464 & 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 518 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 597 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 617 & 0.3 & 0 & 0 & 0 & 0.7 & 0 & 0 \end{array}\right)$$

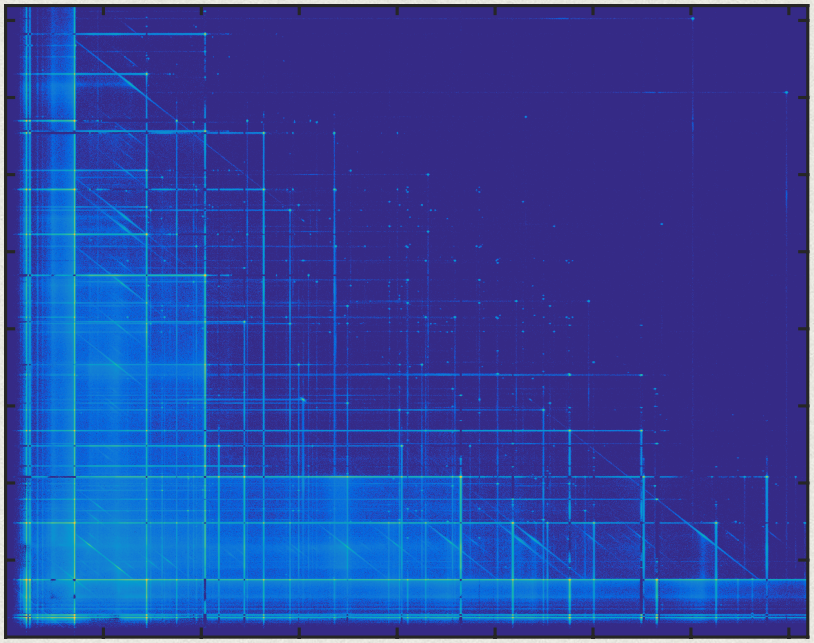
TRANSITION BINNED DATA

In order to use matrix operations, we need data where each row or column of the matrix corresponds to a single transition

Energy Binned
γ-γ Coincidence Matrix

Fitting

Transition Binned
γ-γ Coincidence Matrix



$$C = \left(\begin{array}{c|ccccccc} keV & 213 & 305 & 444 & 464 & 518 & 597 & 617 \\ \hline 213 & 0 & 17748 & 1044 & 5620 & 0 & 3550 & 3845 \\ 305 & 17748 & 0 & 864 & 4651 & 0 & 2938 & 3182 \\ 444 & 1044 & 864 & 0 & 0 & 1512 & 0 & 2040 \\ 464 & 5620 & 4651 & 0 & 0 & 8140 & 7752 & 0 \\ 518 & 0 & 0 & 1512 & 8140 & 0 & 5141 & 5569 \\ 597 & 3550 & 2938 & 0 & 7752 & 5141 & 0 & 0 \\ 617 & 3845 & 3182 & 2040 & 0 & 5569 & 0 & 0 \end{array}\right)$$

DIRECTED COINCIDENCES

Use simulated annealing (or another optimization method) to find D

$$A = I - (S^{-1}DE^{-1} + I)^{-1}$$

$$C = D + D^T$$

Use a cost function to quantify how physically reasonable a level scheme is

$C =$

keV	213	305	444	464	518	597	617
213	0	17748	1044	5620	0	3550	3845
305	17748	0	864	4651	0	2938	3182
444	1044	864	0	0	1512	0	2040
464	5620	4651	0	0	8140	7752	0
518	0	0	1512	8140	0	5141	5569
597	3550	2938	0	7752	5141	0	0
617	3845	3182	2040	0	5569	0	0

Represent candidate solutions as transition orderings, sorted by source excitation energy

For the source excitation energy ordering:

305 , 213, 518, 464, 617, 444, 597 (keV)

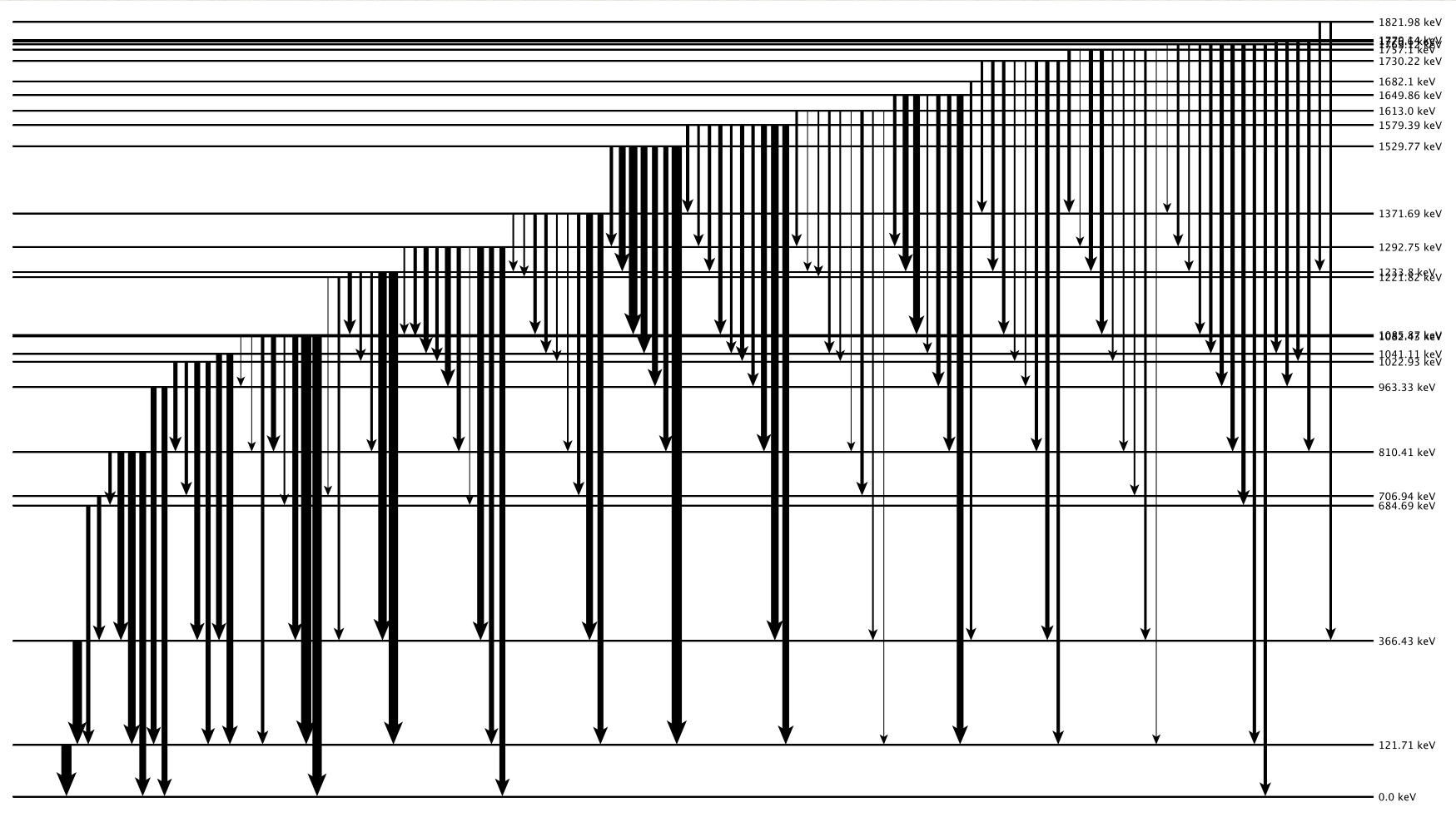
Since transitions decay to lower excitation energies, we can use a source excitation energy ordering to get D from C

$D =$

keV	213	305	444	464	518	597	617
213	0	17748	0	0	0	0	0
305	0	0	0	0	0	0	0
444	1044	864	0	0	1512	0	2040
464	5620	4651	0	0	8140	0	0
518	0	0	0	0	0	0	0
597	3550	2938	0	7752	5141	0	0
617	3845	3182	0	0	5569	0	0

SIMULATED DATA

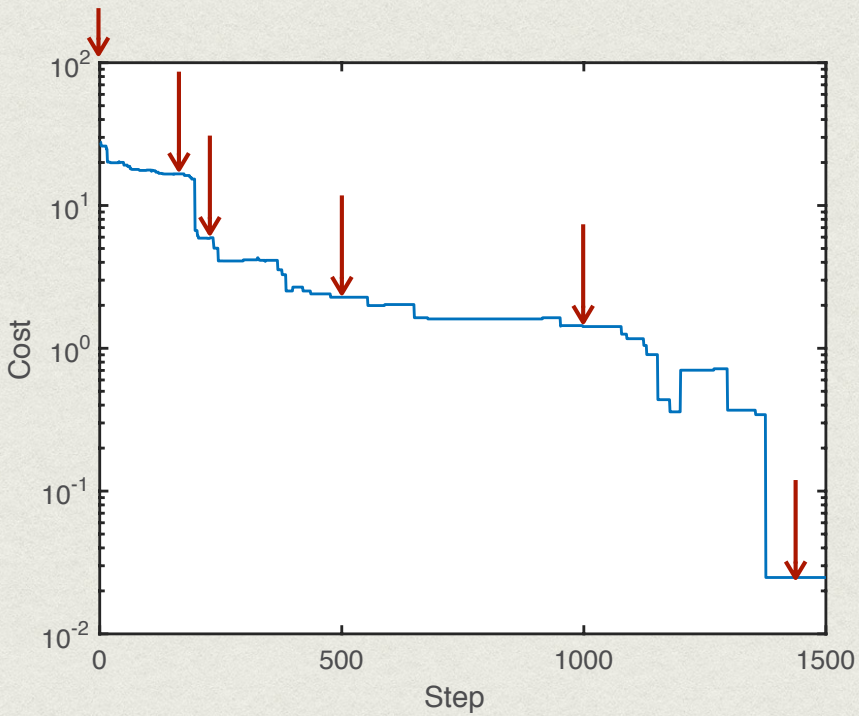
Generated simulated $^{152}Eu^{g}$ decay[1] data with 26 levels and 117 transitions assuming Poisson error in counts with 500 million coincidences

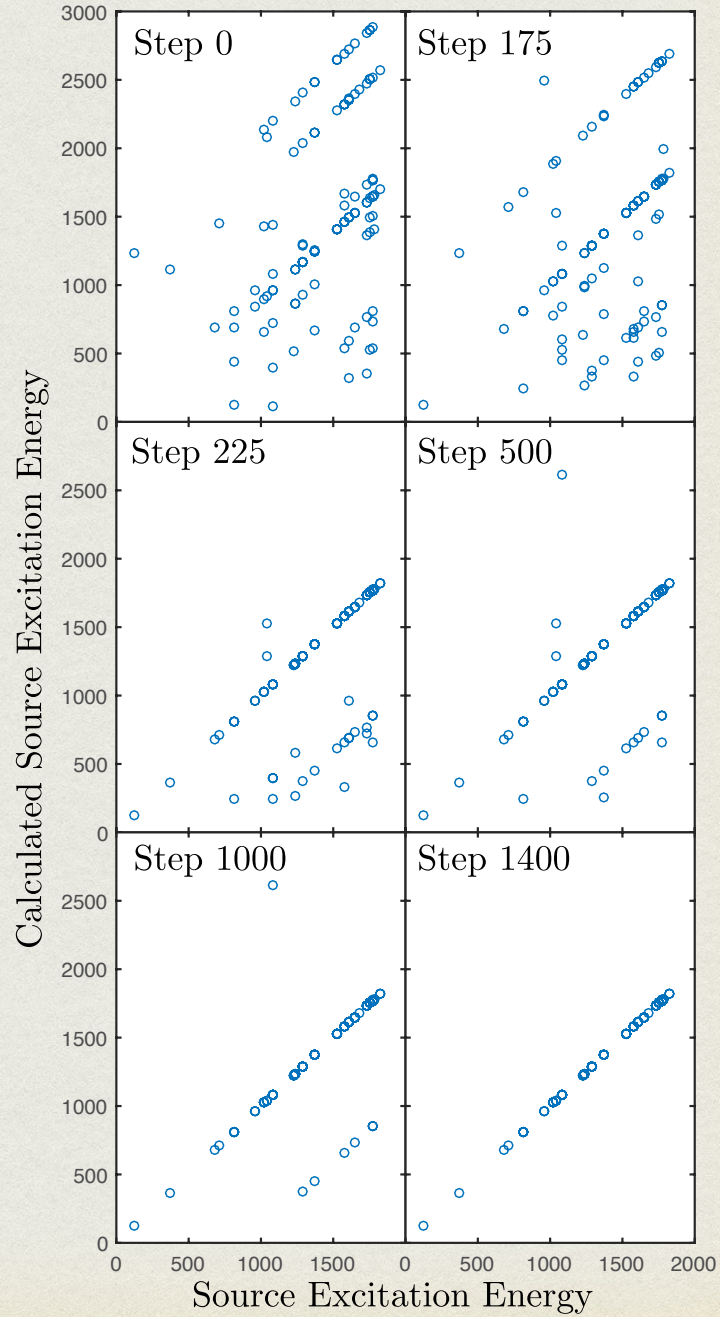


[1] W. D. Kulp. *Phys. Rev. C* **76**, (2007) 034319

RESULTS

Determined the level scheme in ~30 seconds





Simulated $^{152}Eu^{g}$ decay data with 26 levels and 117 transitions