Effective field theory for nuclear vibrations with quantified uncertainties

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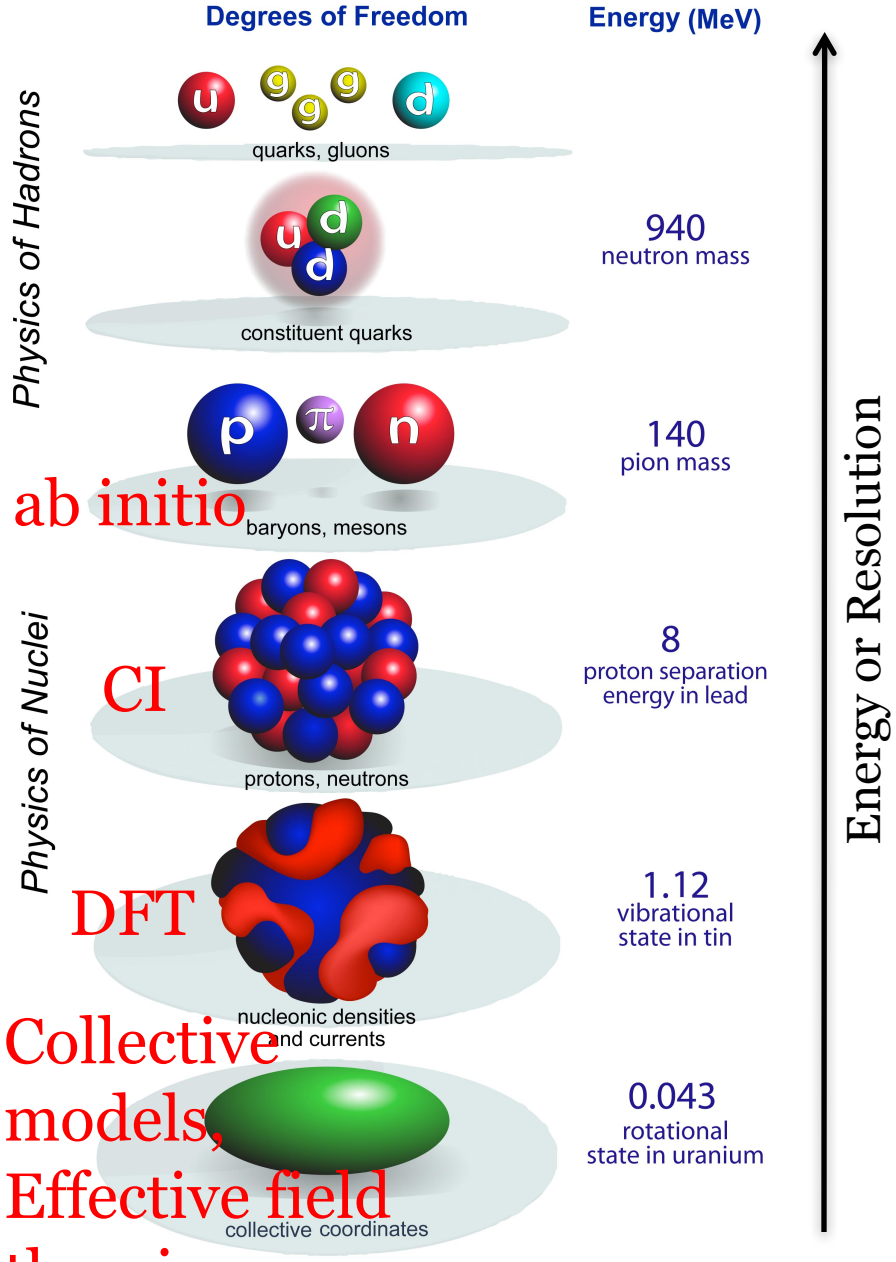
Energy scales and relevant degrees of freedom



Chiral symmetry is broken

Pion is Nambu-Goldstone boson

Tool: Chiral effective field theory

Other EFTs:

Pion-less EFT

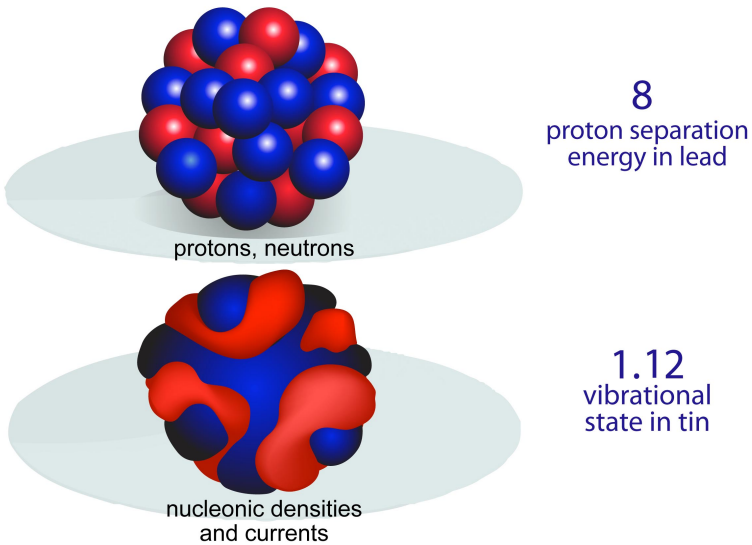
EFT for halo nuclei

EFT for nuclear vibrations

EFT for deformed nuclei

Fig.: Bertsch, Dean, Nazarewicz, SciDAC review (2007)

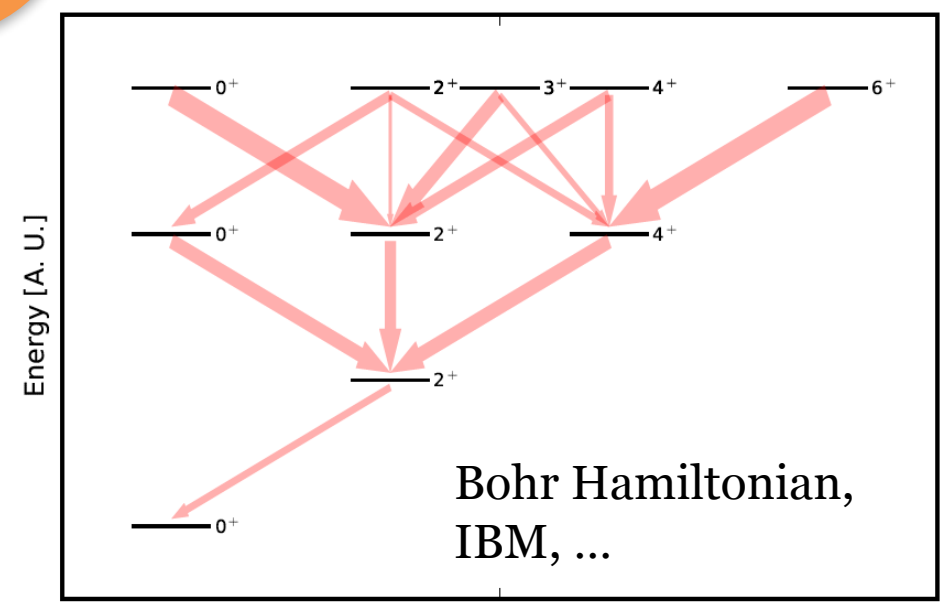
EFT for nuclear vibrations



EFT for nuclear vibrations
[Coello Pérez & TP, PRC (2015)]

While spectra of certain nuclei appear to be harmonic, B(E2) transitions do not.

Garrett & Wood (2010): "Where are the quadrupole vibrations in atomic nuclei?"



Spectrum and B(E2) transitions of the *harmonic* quadrupole oscillator

EFT for nuclear vibrations

Status:

Collective and algebraic models have long been used to describe data in heavy nuclei

Some models "fail" to describe data
But: How does one really know in the absence of theoretical uncertainties?

Continued interest and new data for once-thought vibrational nuclei [e.g. many talks this meeting; Stuchbery 2016]

Aim: Re-examine nuclear vibrations in an EFT

- Systematic and model-independent approach
- Quantify of theoretical uncertainties lacking for models but possible for EFTs
- EFTs can provide us with relations between observables rather than model parameters

Effective field theory for nuclear vibrations

Recipe:

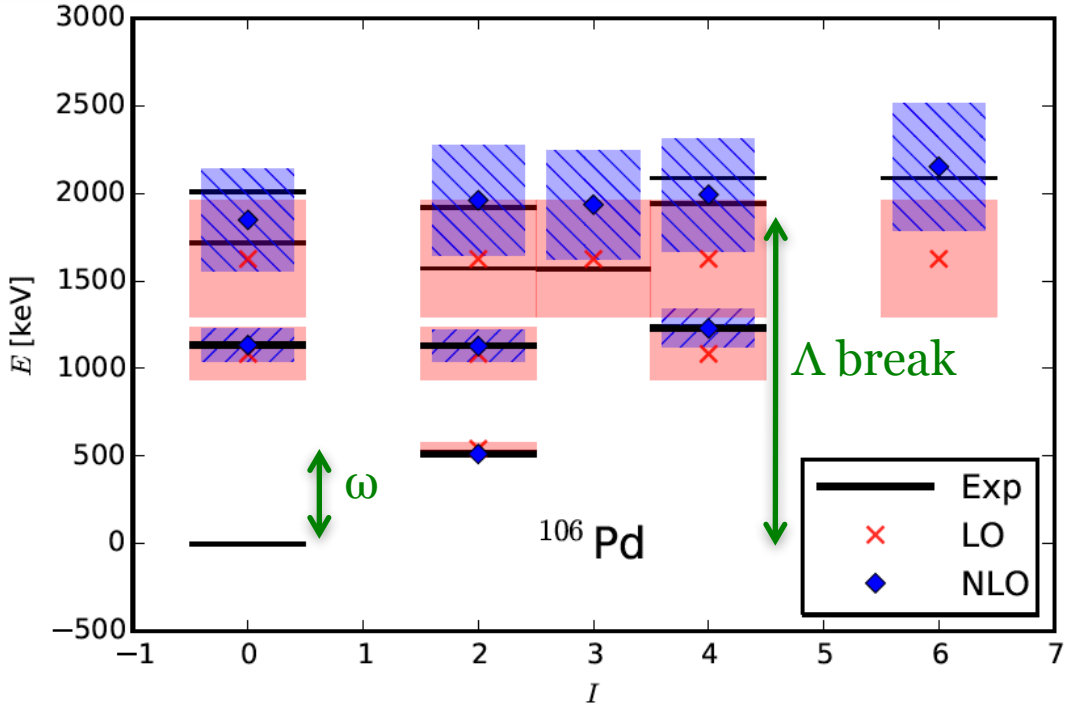
1. Determine low-energy degrees of freedom from spectra: quadrupole bosons
2. Determine relevant symmetries: rotational invariance, parity
3. Write down most general Hamiltonian consistent with symmetries
4. Introduce power counting and order contributions in size

5. Expand other observables in similar way
6. Quantify uncertainties via power counting and Bayesian statistics

EFT for nuclear vibrations

EFT ingredients:

- quadrupole degrees of freedom
- breakdown scale around three-phonon levels
- "small" expansion parameter: ratio of vibrational energy to breakdown scale: $\omega/\Lambda \approx 1/3$



- Uncertainties show 68% DOB intervals from truncating higher EFT orders [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
  - Expand observables according to power counting
  - Employ "naturalness" assumptions as log-normal priors in Bayes' theorem
  - Compute distribution function of uncertainties due to EFT truncation
  - Compute degree-of-believe (DOB) intervals.

Hamiltonian

LO Hamiltonian $\hat{H}_{\mathrm{LO}} = \omega \hat{N}$

NLO correction $\hat{h}_{\mathrm{NLO}} = g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_I \hat{I}^2$

with
$$\hat{N}^2 = (d^\dagger \cdot \tilde{d})^2,$$
$$\hat{\Lambda}^2 = -(d^\dagger \cdot d^\dagger)(\tilde{d} \cdot \tilde{d}) + \hat{N}^2 - 3\hat{N},$$
$$\hat{I}^2 = 10(d^\dagger \otimes \tilde{d})^{(1)} \cdot (d^\dagger \otimes \tilde{d})^{(1)}.$$

"Small" expansion parameter $\varepsilon \equiv (N\omega/\Lambda)$

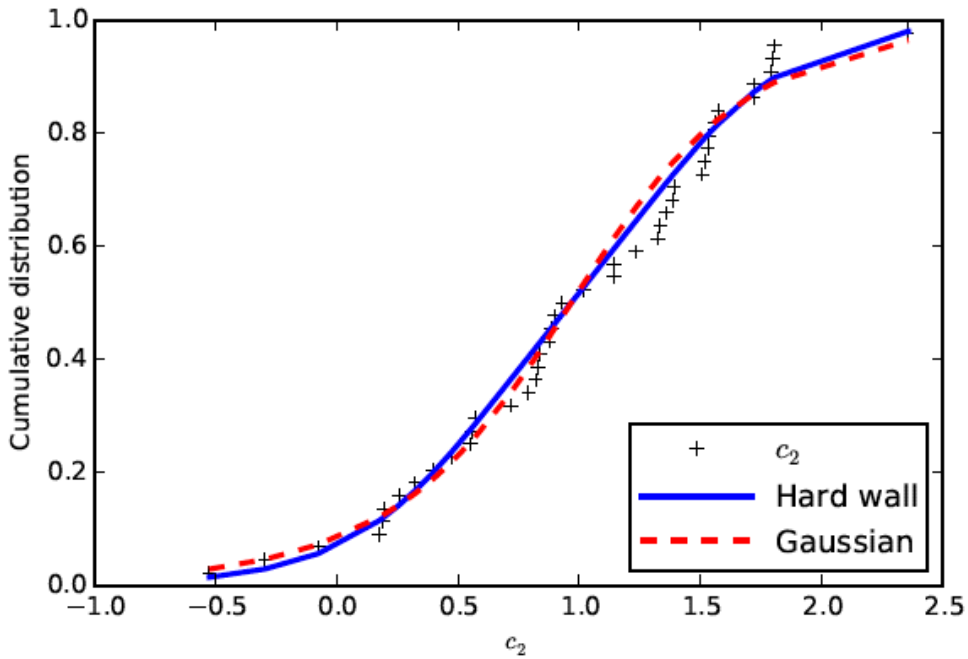
Uncertainty quantification

Furnstahl, Klco, Phillips & Wesolowski (2015)

$$E_{\rm NLO} = \omega N + g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)$$

$$X = X_0 \sum_{n=0}^{\infty} c_n \varepsilon^n$$

$$c_2 \equiv c_2(N, v, I) = \frac{g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)}{\varepsilon^2 \omega}$$



Linear combinations of LECs enter observables. LECs are random, but with EFT expectations, i.e. log-normal distributed. Making assumptions about these distributions then allows one to quantify uncertainties. The assumptions can be tested.

$$\Delta_k^{(M)} = \sum_{n=k+1}^{k+M} c_n \varepsilon^n$$

$$p_M(\Delta | c_0, \ldots, c_k) = \frac{\int_0^\infty {\rm d}c \; {\rm pr}(c) p_M(\Delta|c) \prod_{m=0}^{k} {\rm pr}(c_m|c)}{\int_0^\infty {\rm d}c \; {\rm pr}(c) \prod_{m=0}^{k} {\rm pr}(c_m|c)}$$

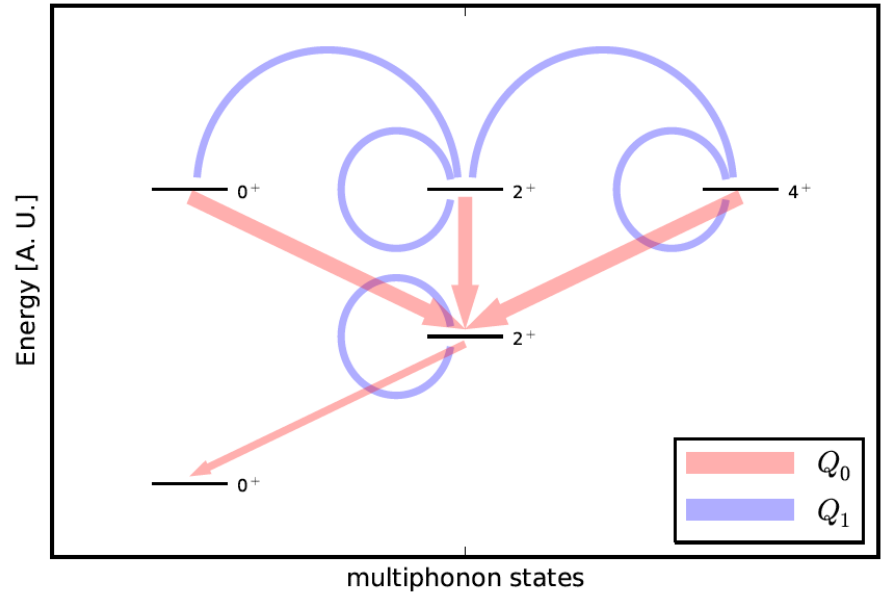
Main EFT result: Sizeable quadrupole matrix elements

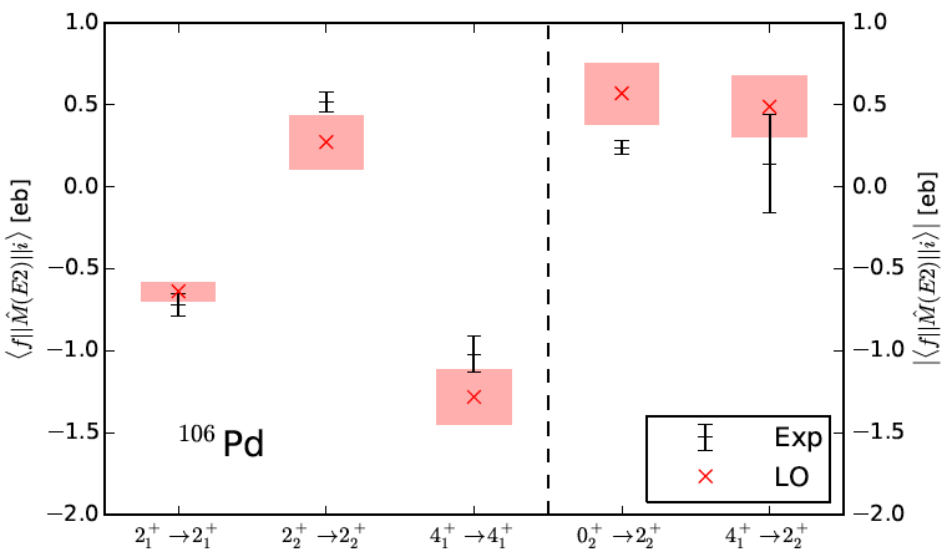
In the EFT, the quadrupole operator must also be expanded:

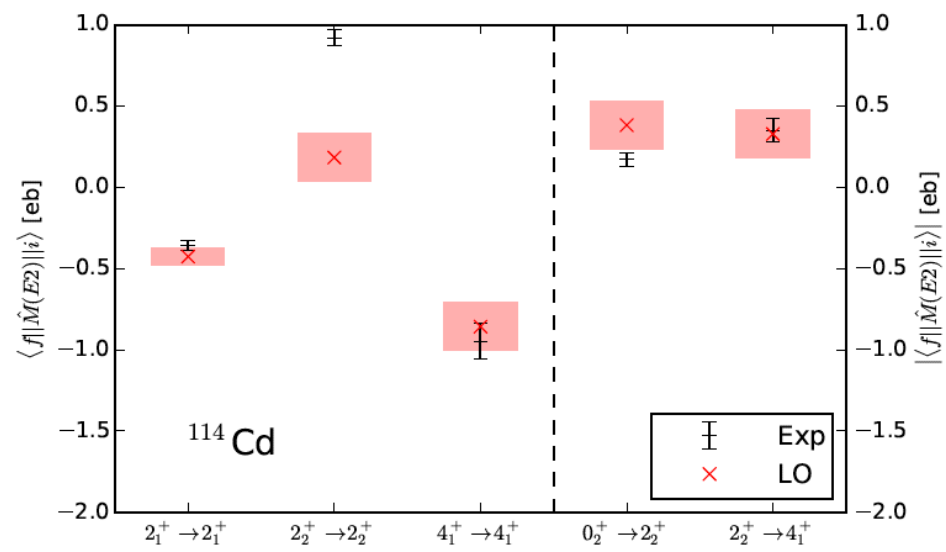
$$\hat{Q}_\mu = Q_0 \left(d^\dagger_\mu + \tilde{d}_\mu \right) + Q_1 \left(d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d} + 2 d^\dagger \times \tilde{d} \right)^{(2)}_\mu$$

Subleading corrections are sizable:

$$Q_1 \sim \left(\frac{\omega}{\Lambda} \right)^{1/2} Q_0$$





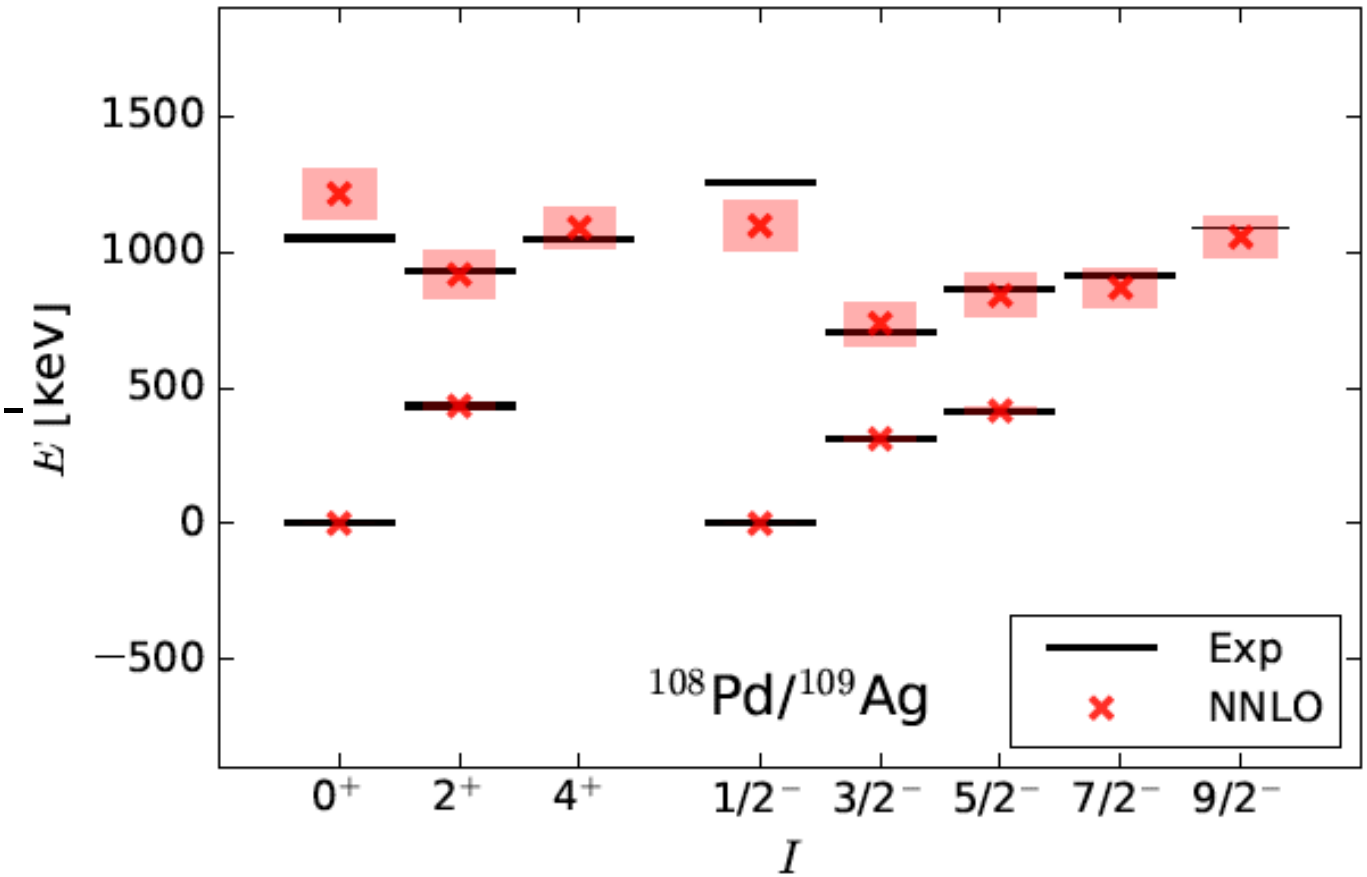


Work in progress: Fermion coupled to vibrating nucleus

Idea: In the spirit of Halo EFT [Bertulani, Hammer, van Kolck (2002); Higa, Hammer, van Kolck (2008); Hammer & Philipps (2011); Ryberg et al. (2014)], add a fermion to describe odd-mass neighbors

First step:
EFT for j=1/2 fermion (or fermion hole) coupled to even-even core.

Two new LECs enter at LO



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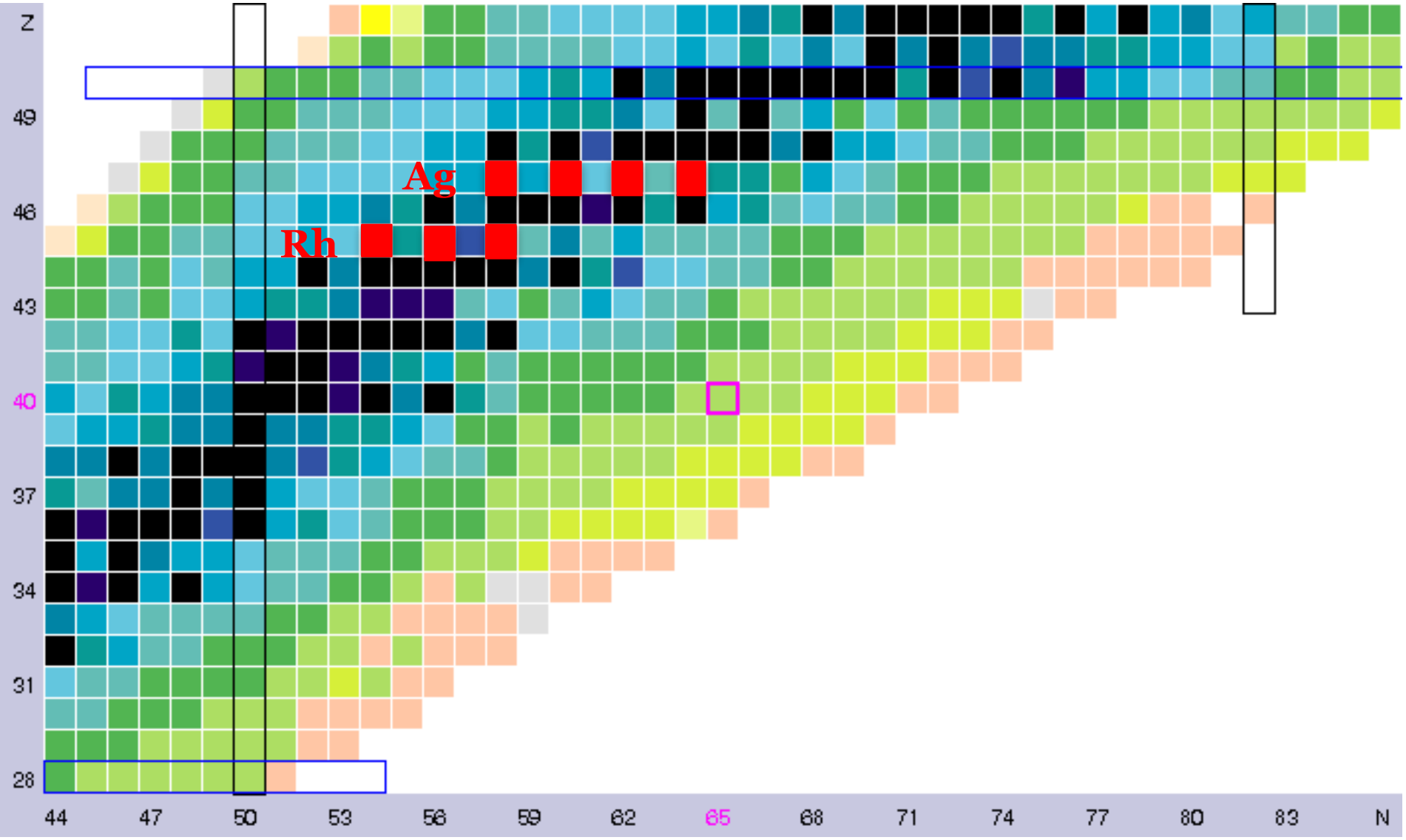
^{99,101,103}Rh and ^{105,107,109,111}Ag as a proton coupled to ^{98,100,102}Ru and ^{104,106,108,110}Pd, respectively, or ^{107,109,111}Ag as a proton-hole in ^{108,110,112}Cd



Static E2 moments (in eb)

Nucleus	I_i^π	$Q_{\rm exp}$	$Q_{\rm EFT}$
^{102}Ru	2_1^+	$-0.63(3)$	$-0.41(6)$
	2_2^+		$0.18(18)$
	4_1^+		$-0.82(14)$
^{103}Rh	$\frac{3}{2}_1^-$	$-0.3(2)$	$-0.29(7)$
	$\frac{5}{2}_1^-$	$-0.4(2)$	$-0.41(6)$
^{106}Pd	2_1^+	$-0.54(4)$	$-0.50(7)$
	2_2^+	$0.39(6)$	$0.21(20)$
	4_1^+	$-0.79(11)$	$-1.00(17)$
^{107}Ag	$\frac{3}{2}_1^-$		$-0.35(8)$
	$\frac{5}{2}_1^-$		$-0.50(7)$

Nucleus	I_i^π	$Q_{\rm exp}$	$Q_{\rm EFT}$
^{108}Pd	2_1^+	$-0.56(3)$	$-0.57(7)$
	2_2^+	$0.73(9)$	$0.24(20)$
	4_1^+	$-0.78(11)$	$-1.14(17)$
^{109}Ag	$\frac{3}{2}_1^-$	$-0.7(3)$	$-0.40(8)$
	$\frac{5}{2}_1^-$	$-0.3(3)$	$-0.57(6)$
^{110}Cd	2_1^+	$-0.39(3)$	$-0.57(7)$
	2_2^+		$0.24(17)$
	4_1^+		$-1.12(14)$
^{109}Ag	$\frac{3}{2}_1^-$	$-0.7(3)$	$-0.39(6)$
	$\frac{5}{2}_1^-$	$-0.3(3)$	$-0.56(6)$

Single LEC fit to all data with EFT weighting.

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E2 transition strengths

Predictions for transitions that change phonon numbers (in Weisskopf units).

$$\hat{Q}_\mu = Q_0 \left(d^\dagger_\mu + \tilde{d}_\mu \right)$$

Single LEC adjusted to data

Nucleus	$I_i^\pi \to I_f^\pi$	$B(E2)_{\rm exp}$	$B(E2)_{\rm EFT}$
^{110}Cd	$2_1^+ \to 0_1^+$	27(1)	23(8)
	$0_2^+ \to 2_1^+$		46(15)
	$2_2^+ \to 2_1^+$	30(5)	46(15)
	$4_1^+ \to 2_1^+$	42(9)	46(15)
^{109}Ag	$\frac{3}{2}^-_1 \to \frac{1}{2}^-_1$	40(40)	23(8)
	$\frac{5}{2}^-_1 \to \frac{1}{2}^-_1$	41(6)	23(8)
	$\frac{1}{2}^-_2 \to \frac{3}{2}^-_1$		19(16)
	$\frac{1}{2}^-_2 \to \frac{5}{2}^-_1$		28(16)
	$\frac{3}{2}^-_2 \to \frac{3}{2}^-_1$	49(24)	33(16)
	$\frac{3}{2}^-_2 \to \frac{5}{2}^-_1$		14(16)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_1$	8(4)	9(16)
	$\frac{5}{2}^-_2 \to \frac{5}{2}^-_1$	10(7)	37(16)
	$\frac{7}{2}^-_1 \to \frac{3}{2}^-_1$		42(16)
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$		5(16)
	$\frac{9}{2}^-_1 \to \frac{5}{2}^-_1$		47(16)

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Magnetic moments: Relations between even-even and even-odd nuclei

Nucleus	I_i^π	$\mu_{\rm exp}(I_i^\pi)$	$\mu_{\rm EFT}(I_i^\pi)$
^{102}Ru	2_1^+	0.85(3)*	0.85(5)
	2_2^+		0.85(10)
	4_1^+		1.70(8)
^{103}Rh	$\frac{1}{2}_1$	−0.088*	−0.088
	$\frac{3}{2}_1$	0.77(7)	0.81(5)
	$\frac{5}{2}_1$	1.08(4)	0.76(5)
	$\frac{7}{2}_1$	2.0(6)	1.7(1)
	$\frac{9}{2}_1$	2.8(5)	1.6(1)

Nucleus	I_i^π	$\mu_{\rm exp}(I_i^\pi)$	$\mu_{\rm EFT}(I_i^\pi)$
^{106}Pd	2_1^+	0.79(2)*	0.79(5)
	2_2^+	0.71(10)	0.79(10)
	4_1^+	1.8(4)	1.58(8)
^{107}Ag	$\frac{1}{2}_1$	−0.11*	−0.11
	$\frac{3}{2}_1$	0.98(9)	0.78(5)
	$\frac{5}{2}_1$	1.02(9)	0.68(4)
	$\frac{7}{2}_1$		1.6(1)
	$\frac{9}{2}_1$		1.5(1)

At LO, one new LEC enters to describe odd-mass neighbor.
Results in units of nucleon magnetons.

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M1 transition strengths

Nucleus	$I_i^\pi \to I_f^\pi$	$B(M1)_{\rm exp}$	$B(M1)_{\rm EFT}$
^{103}Rh	$\frac{5}{2}^-_1 \to \frac{3}{2}^-_1$		0.034(2)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_2$		0.034(5)
	$\frac{9}{2}^-_1 \to \frac{7}{2}^-_1$		0.038(3)
^{107}Ag	$\frac{5}{2}^-_1 \to \frac{3}{2}^-_1$	0.033(4)	0.036(2)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_2$		0.036(4)
	$\frac{9}{2}^-_1 \to \frac{7}{2}^-_1$		0.040(2)
^{109}Ag	$\frac{5}{2}^-_1 \to \frac{3}{2}^-_1$	0.043(7)	0.036(2)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_2$		0.036(3)
	$\frac{9}{2}^-_1 \to \frac{7}{2}^-_1$		0.040(2)

Predictions for transitions that conserve phonon numbers (in Weisskopf units).

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M1 transition strengths

Nucleus	$I_i^\pi \to I_f^\pi$	$B(M1)_{\rm exp}$	$B(M1)_{\rm EFT}$
^{103}Rh	$\frac{3}{2}^-_1 \to \frac{1}{2}^-_1$	0.12(1)	0.12(1)
	$\frac{1}{2}^-_2 \to \frac{3}{2}^-_1$		0.032(36)
	$\frac{3}{2}^-_2 \to \frac{3}{2}^-_1$		0.027(18)
	$\frac{3}{2}^-_2 \to \frac{5}{2}^-_1$		0.015(18)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_1$	0.014(2)	0.010(12)
	$\frac{5}{2}^-_2 \to \frac{5}{2}^-_1$	0.020(3)	0.020(12)
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$		0.072(9)
^{109}Ag	$\frac{3}{2}^-_1 \to \frac{1}{2}^-_1$	0.117(15)	0.117(9)
	$\frac{1}{2}^-_2 \to \frac{3}{2}^-_1$		0.031(35)
	$\frac{3}{2}^-_2 \to \frac{3}{2}^-_1$	0.16(7)	0.00(2)
	$\frac{3}{2}^-_2 \to \frac{5}{2}^-_1$		0.034(17)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_1$	0.036(16)	0.023(12)
	$\frac{5}{2}^-_2 \to \frac{5}{2}^-_1$	0.10(4)	0.11(1)
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$		0.070(9)

Results for transitions that change phonon numbers (in Weisskopf units).

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Magnetic moments of Cd

Data from Stuchbery 2016.

Extend particle-vibrator EFT to two-fermion orbitals.

Comparison with particle-rotor model of Stuchbery.

(in units of nucleon magnetons)

Nucleus	I_i^π	$\mu_{\rm PR}(I_i^\pi)$	$\mu_{\rm exp}(I_i^\pi)$	$\mu_{\rm EFT}(I_i^\pi)$
^{110}Cd	2_1^+		0.52(4)*	0.52(14)
	2_2^+			0.52(28)
	4_1^+			1.0(2)
^{111}Cd	$\frac{1}{2}_1$	−0.62	−0.59*	−0.59
	$\frac{3}{2}_1$	0.9	0.9(6)	0.8(1)
	$\frac{5}{2}_2$	0.8	0.5(1)	−0.07(14)
^{112}Cd	2_1^+		0.64(16)*	0.64(15)
	2_2^+			0.64(30)
	4_1^+			1.3(2)
^{113}Cd	$\frac{1}{2}_1$	−0.56	−0.62*	−0.62
	$\frac{5}{2}_1$		−0.77*	−0.77
	$\frac{3}{2}_1$	0.8	−0.6(10)	0.9(2)
				−0.3(1)
	$\frac{5}{2}_2$	0.65	0.35(10)	0.02(14)
	$\frac{3}{2}_2$	1.2	2.1(6)	0.9(2)

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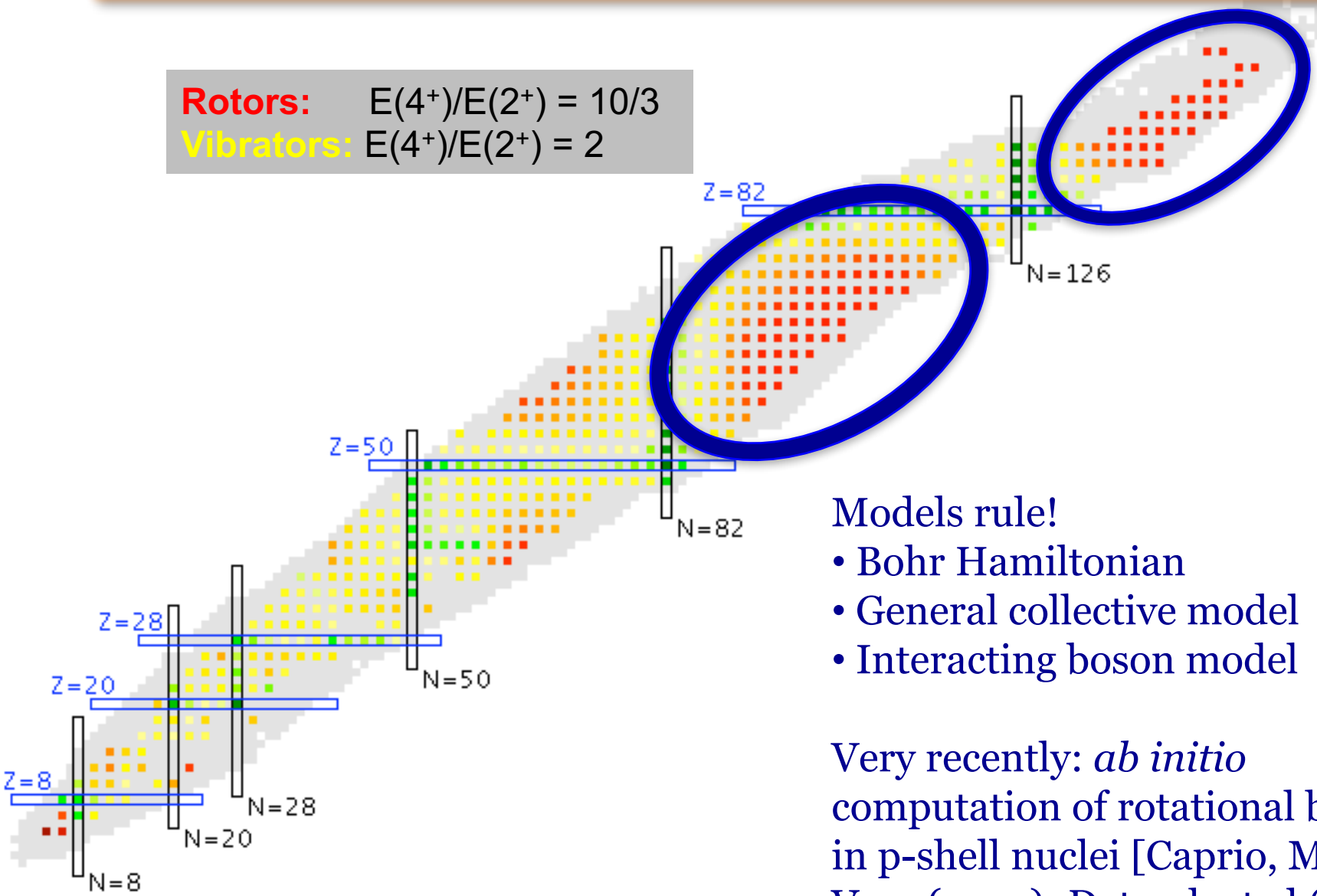
Summary

- From EFTs to nuclei
  - Exploit separation of scales
  - Systematic expansion of Hamiltonians *and* other operators
- EFT for nuclear vibrations
  - Quadrupole moments are of natural size (and sizeable) due to NLO corrections
  - Picture of anharmonic vibrations consistent with data within uncertainties
  - Fermion-vibrator EFT for selected odd-mass nuclei
  - Fair description of wide range of M1 and E2 data
- EFT for deformed nuclei
  - Interband transitions correctly described due to new terms in operator

EFT for deformed nuclei

Rotors: $E(4^+)/E(2^+) = 10/3$
Vibrators: $E(4^+)/E(2^+) = 2$

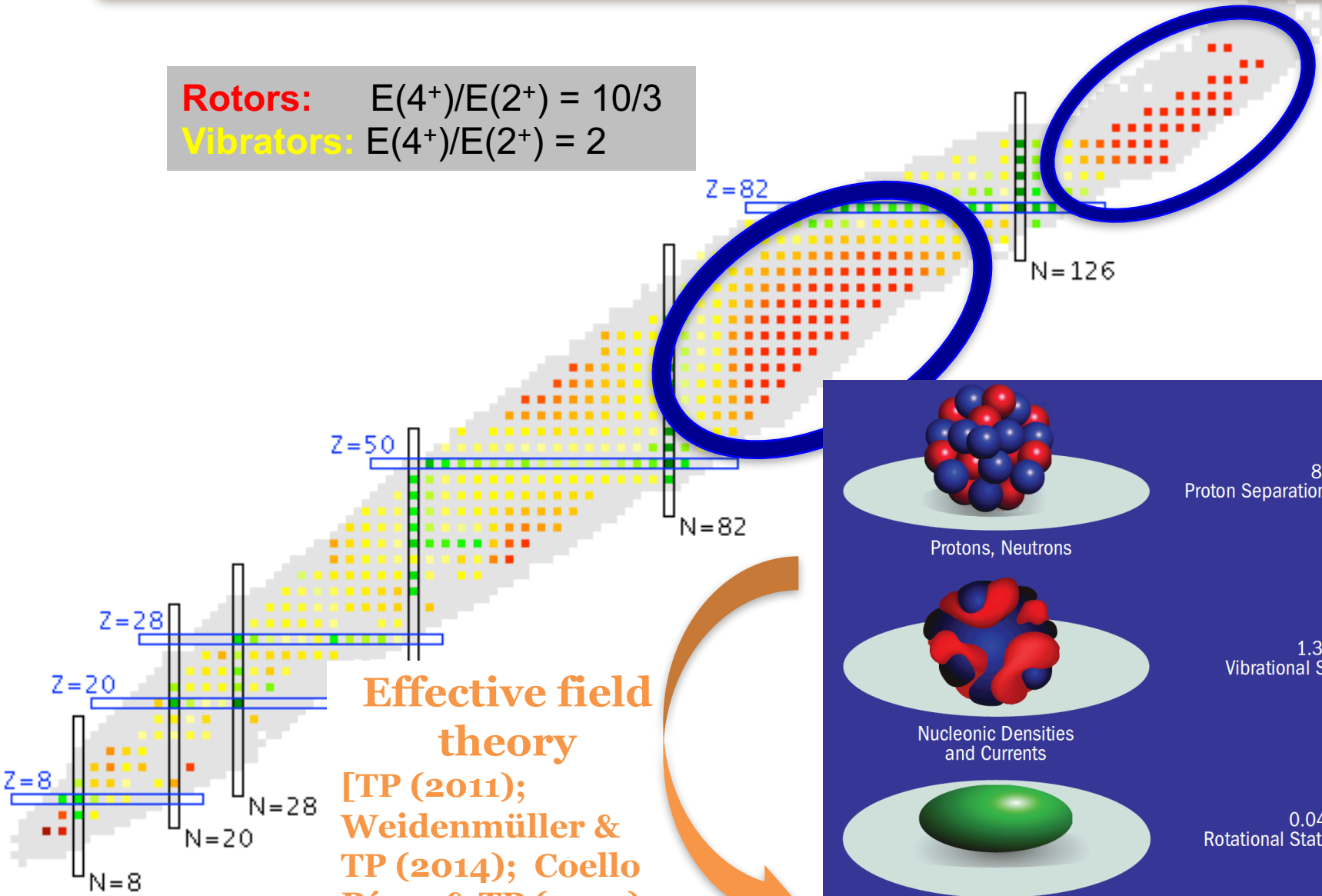


Models rule!
- Bohr Hamiltonian
- General collective model
- Interacting boson model

Very recently: *ab initio* computation of rotational bands in p-shell nuclei [Caprio, Maris, Vary (2013); Dytrych et al (2013)]

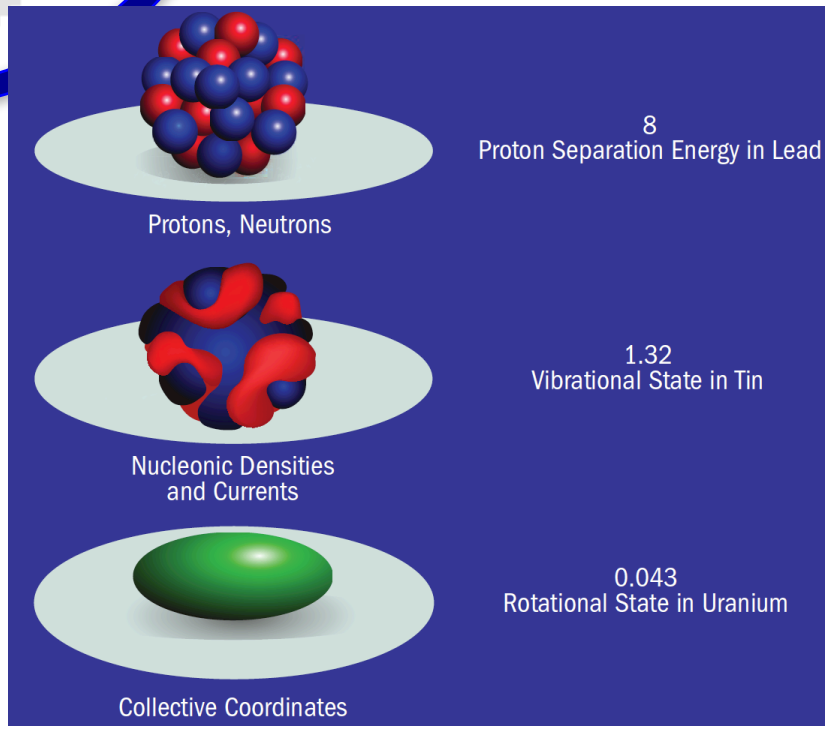
EFT for deformed nuclei

Rotors: $E(4^+)/E(2^+) = 10/3$
Vibrators: $E(4^+)/E(2^+) = 2$

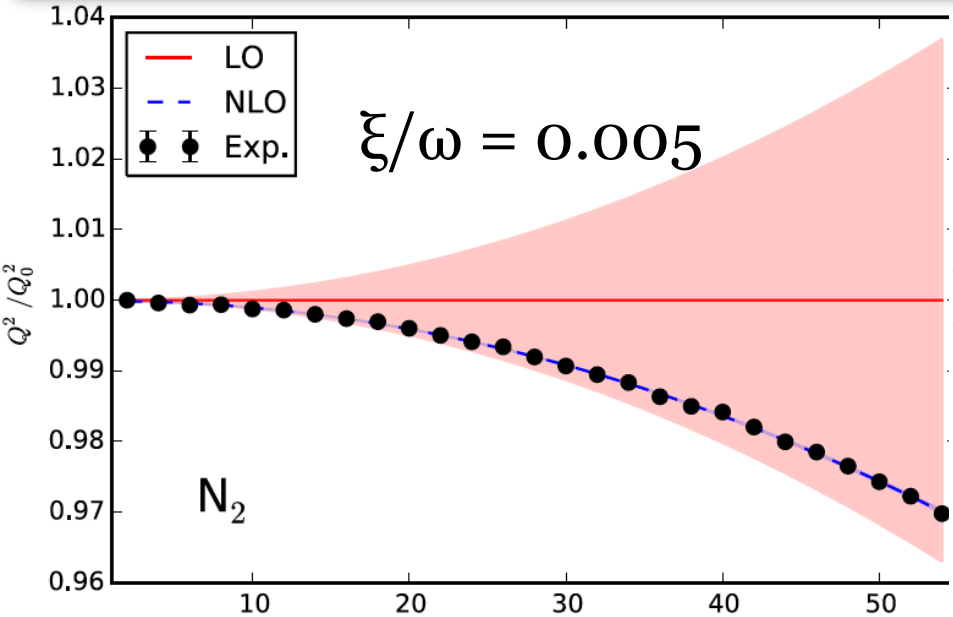


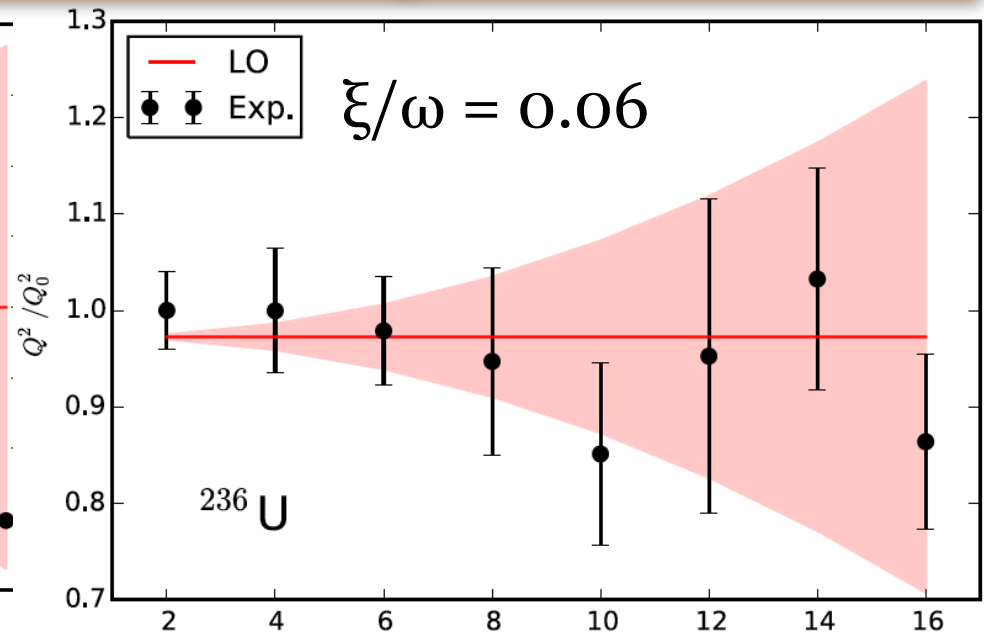
Effective field theory

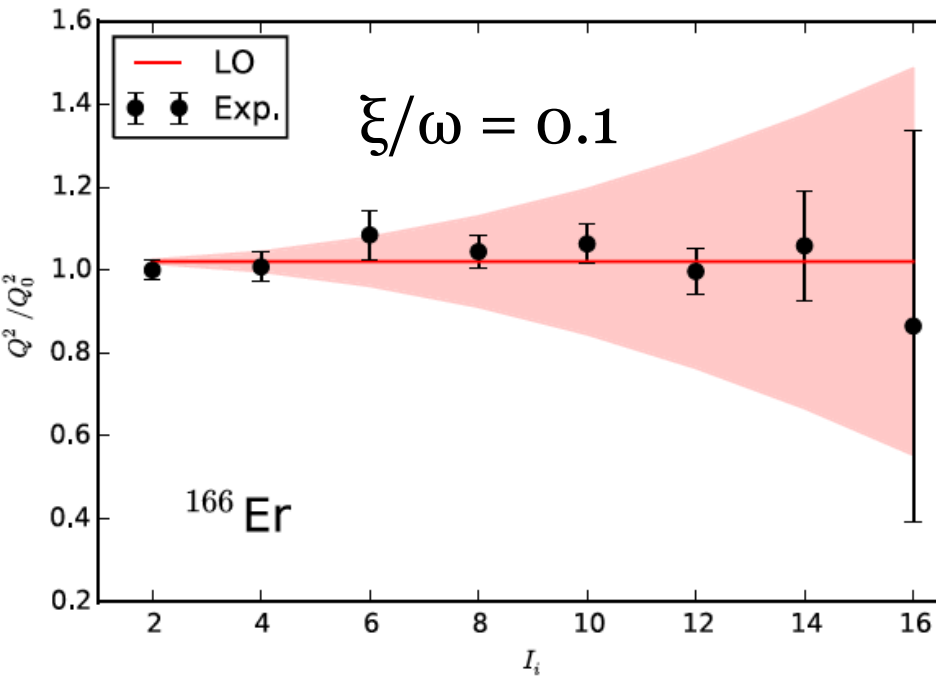
[TP (2011); Weidenmüller & TP (2014); Coello Pérez & TP (2015);

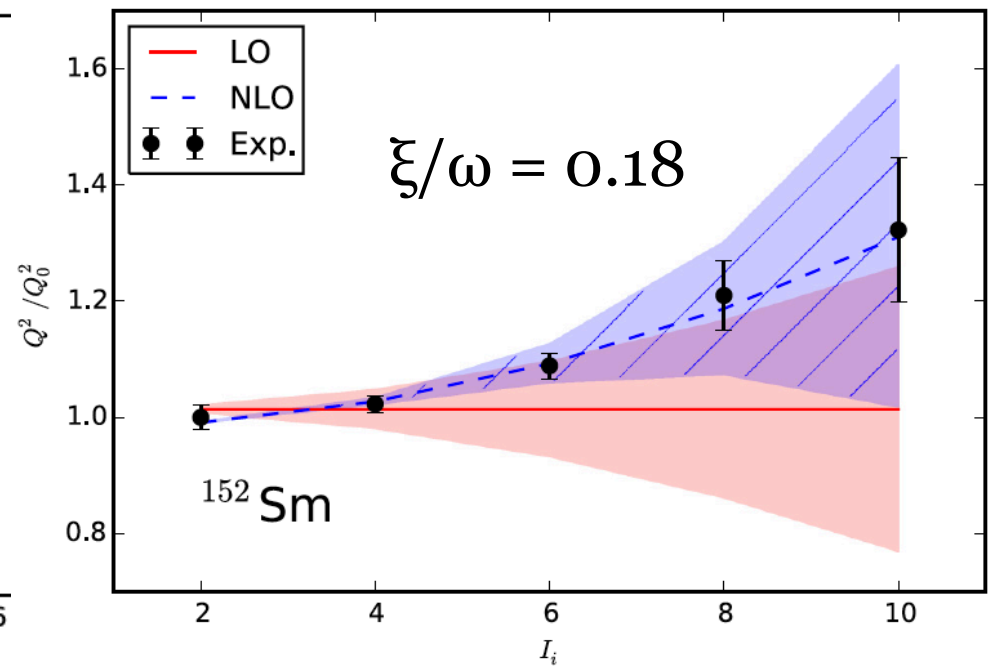


EFT works well for a wide range of rotors



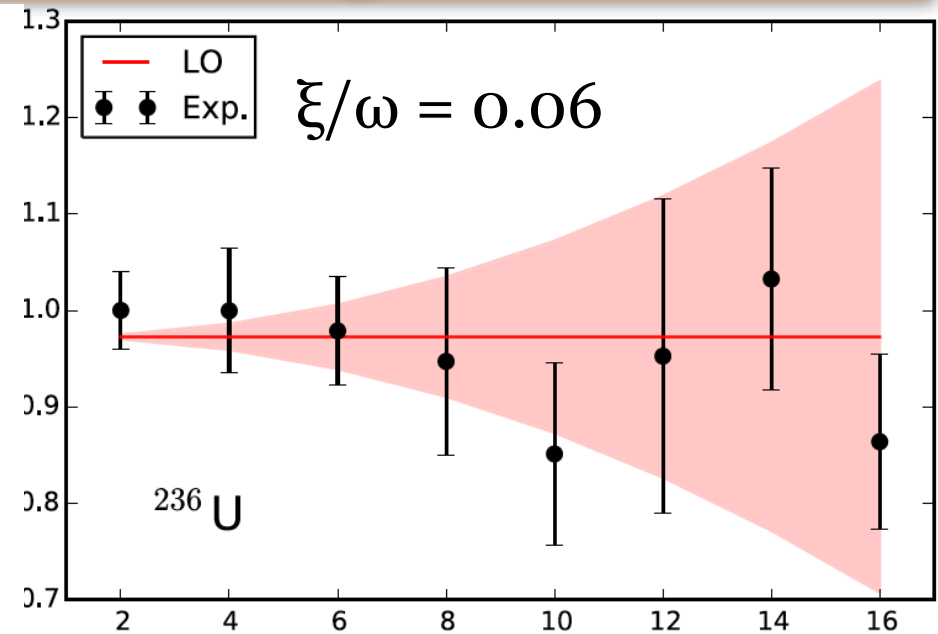


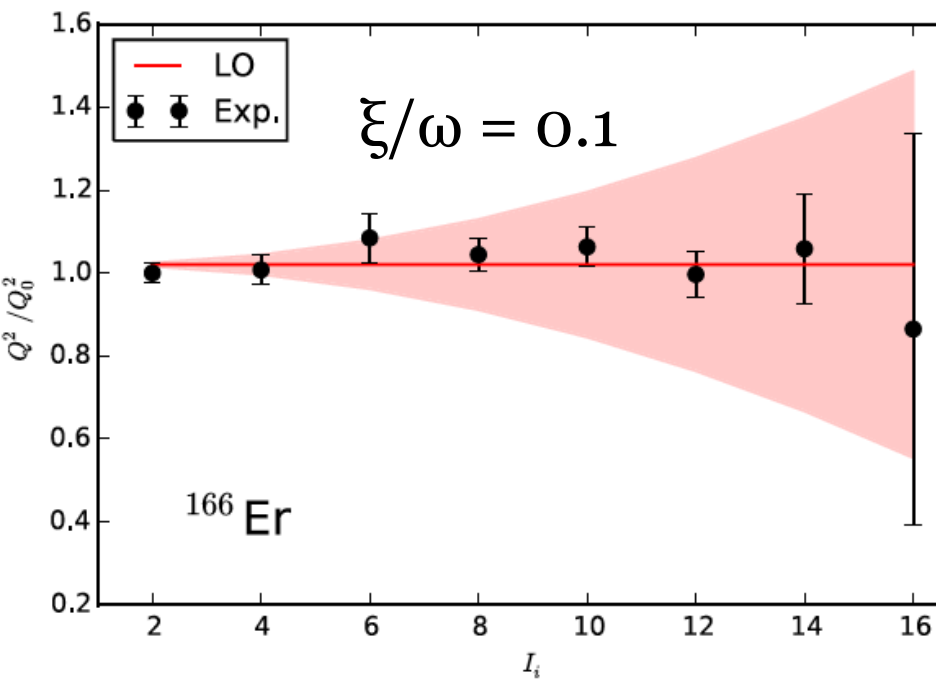


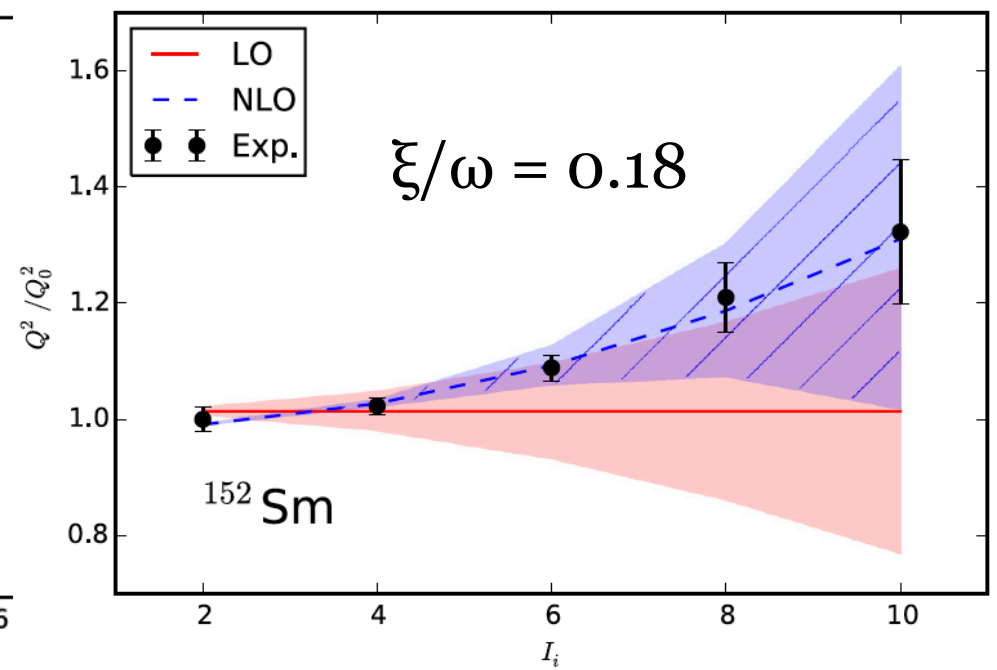


EFT works well for a wide range of rotors

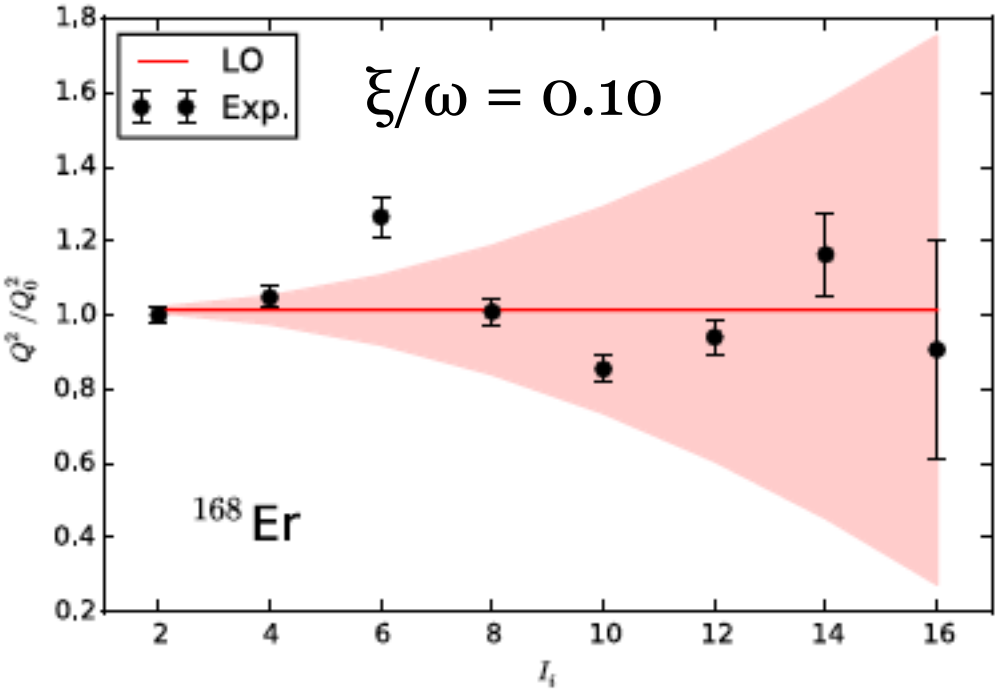
Bohr & Mottelson (1975):
"The accuracy of the present measurements of E2-matrix elements in the ground-state bands of even even nuclei is in most cases barely sufficient to detect deviations from the leading-order intensity relations."







EFT can not explain oscillatory patterns in supposedly "good" rotors ^{168}Er, ^{174}Yb

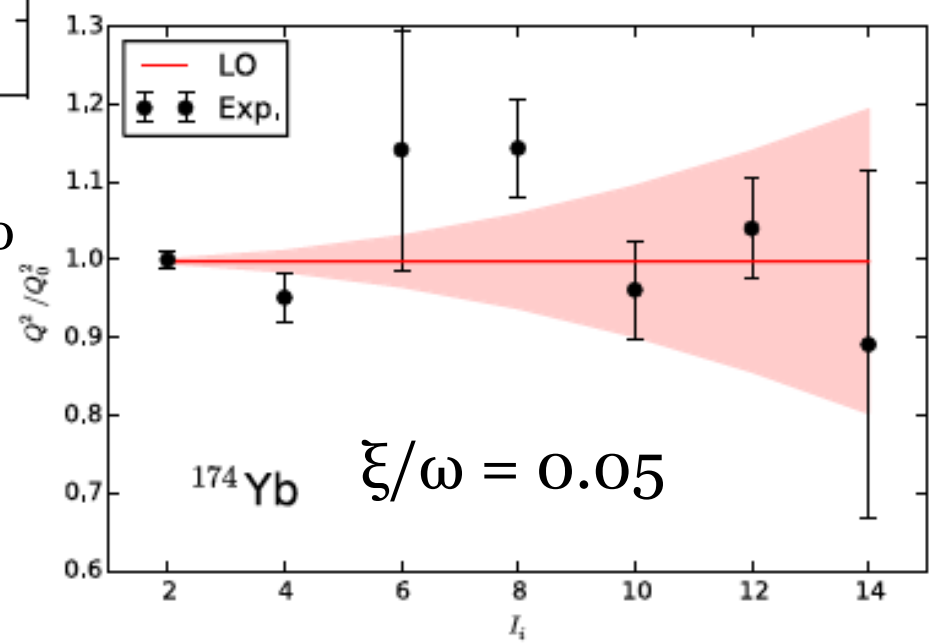


Based on results for molecules, well-deformed nuclei, and transitional nuclei, EFT suggests that a few transitions in text-book rotors could merit re-measurement.

^{168}Er: B(E2) for $6^+ \rightarrow 4^+$ very difficult to understand.

^{174}Yb: B(E2) for $8^+ \rightarrow 6^+$ difficult to reconcile with $4^+ \rightarrow 2^+$.

Theoretical uncertainty estimates relevant.



EFT and weak interband transitions (^{154}Sm)

$i \to f$	$B(E2)_{\rm exp}$	$B(E2)_{\rm ET}$	$B(E2)_{\rm CBS}$	$B(E2)_{\rm BH}$
$2_g^+ \to 0_g^+$	0.863 (5)	0.863[a]	0.853	0.863
$4_g^+ \to 2_g^+$	1.201 (29)	1.233 (9)	1.231	1.234
$6_g^+ \to 4_g^+$	1.417 (39)	1.358 (23)	1.378	1.355
$8_g^+ \to 6_g^+$	1.564 (83)	1.421 (43)	1.471	1.424
$2_\gamma^+ \to 0_g^+$	0.0093 (10)	0.0110 (28)		0.0492
$2_\gamma^+ \to 2_g^+$	0.0157 (15)	0.0157[a]		0.0703
$2_\gamma^+ \to 4_g^+$	0.0018 (2)	0.0008 (2)		0.0050
$2_\beta^+ \to 0_g^+$	0.0016 (2)	0.0025 (6)	0.0024	0.0319
$2_\beta^+ \to 2_g^+$	0.0035 (4)	0.0035[a]	0.0069	0.0456
$2_\beta^+ \to 4_g^+$	0.0065 (7)	0.0063 (16)	0.0348	0.0821

[a]Values employed to adjust the LECs of the effective theory.

In-band transitions [in e^2b^2] are LO, inter-band transitions are NLO. Effective theory is more complicated than Bohr Hamiltonian both in Hamiltonian and E2 transition operator. EFT correctly predicts strengths of inter-band transitions with natural LECs.

[E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)]

Computation of emergent phenomena

Emergent phenomena

- Nuclear saturation
- Nuclear deformation and vibrations
- Clustering (α particles, halos, ...)

Really hard to compute from first principles

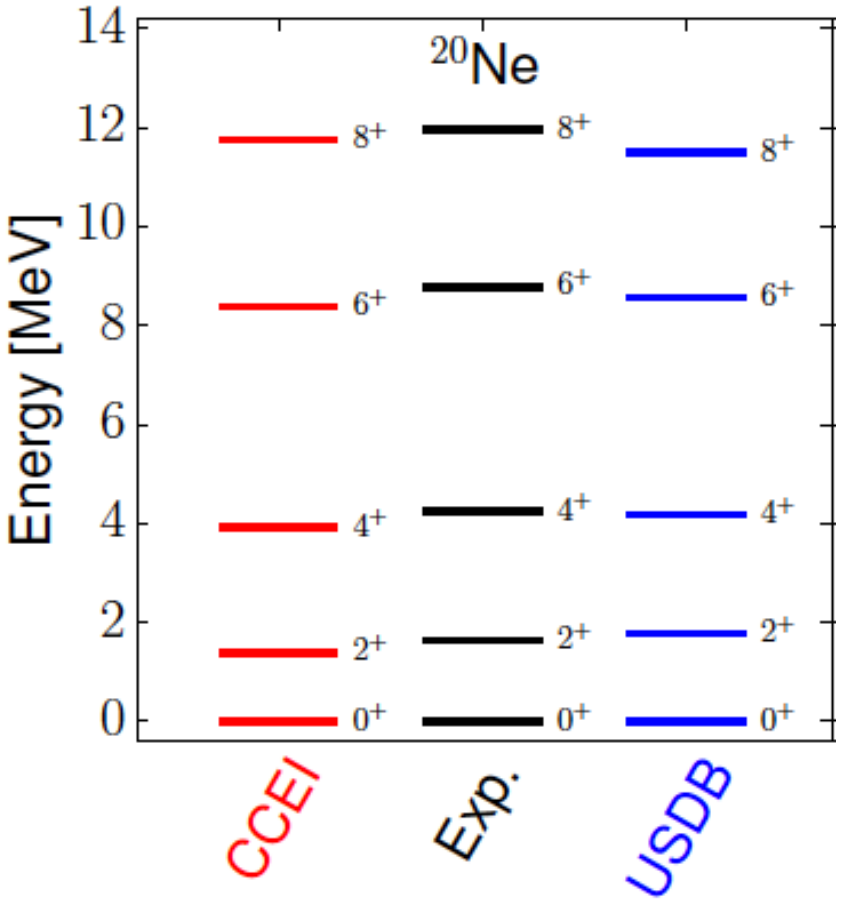
- Finely tuned
- Emergent low-energy scales / multi-scale problem
- Complex and collective in nature

Usually fixed in models

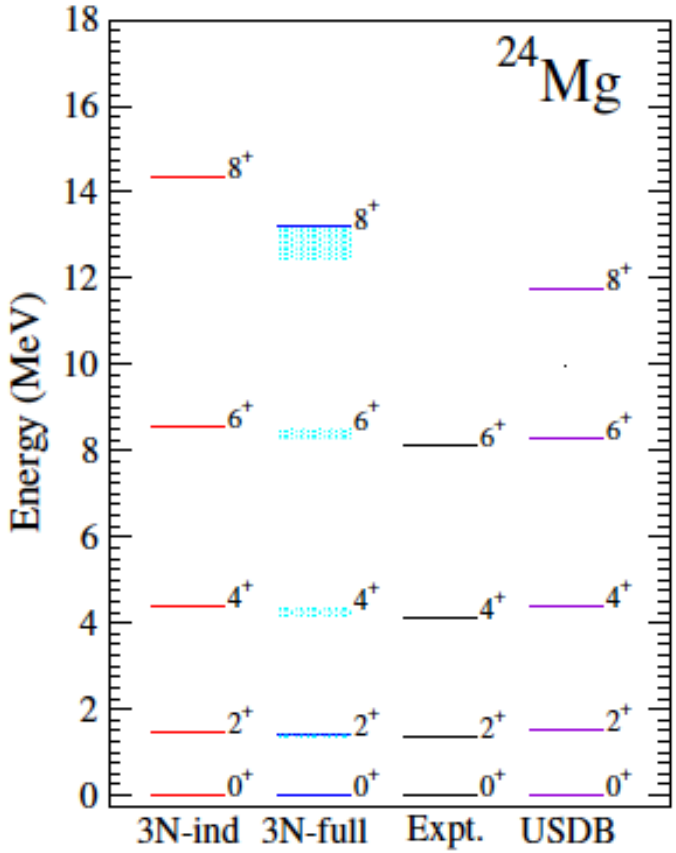
- $\hbar\omega$ sets nuclear saturation & radii in shell model
- Deformed shell model, collective & algebraic models
- α-particle cluster models of the nucleus

Opportunities for EFTs & challenges for ab initio approaches

Nuclear deformation from first principles



Jansen et al., 1511.00757



Stroberg et al., 1511.02802

Deformation in *p*-shell nuclei:

Caprio, Maris & Vary, PLB (2013); Caprio et al., IJMPE (2015); Dytrych et al., PRL (2013)

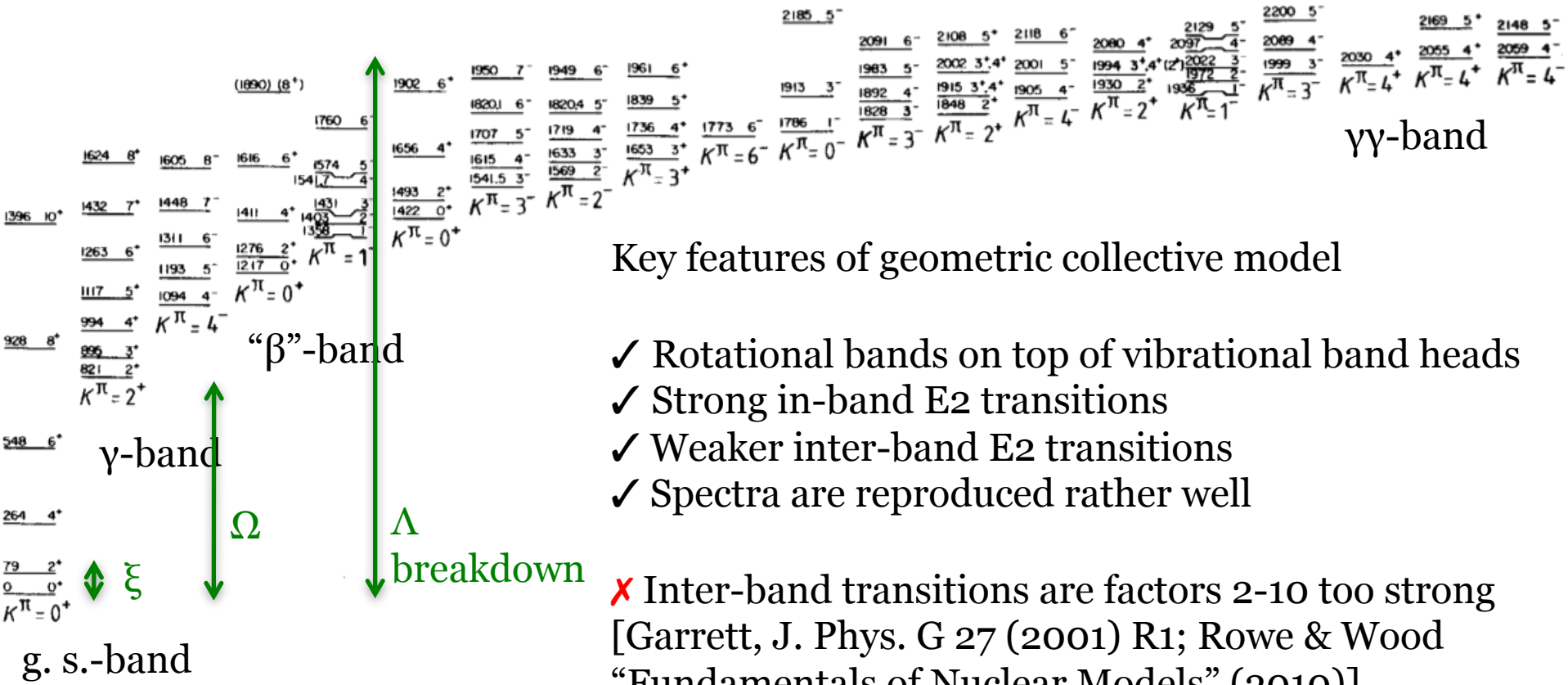
B(E2) transitions in vibrational nuclei

Nucleus	$2_1^+ \to 0_1^+$	EFT	$0_2^+ \to 2_1^+$	$2_2^+ \to 2_1^+$	$4_1^+ \to 2_1^+$	EFT
^{62}Ni	12.1(4)	11(4)	42(23)	14.9(42)	21(6)	21(7)
^{98}Ru	31(1)	28(9)		47(5)	57.6(40)	56(19)
^{100}Ru	35.6(4)	24(8)	35(5)	30.9(4)	51(4)	47(16)
^{106}Pd	44.3(15)	30(10)	35(8)	44(4)	76(11)	61(20)
^{108}Pd	49.5(13)	37(12)	52(5)	71(5)	73(8)	74(25)
^{110}Cd	27.0(8)	21(7)		30(5)	42(9)	42(14)
^{112}Cd	30.2(3)	23(8)	51(14)	15(3)	61(6)	46(15)
^{114}Cd	31.1(19)	22(7)	27.4(17)	22(6)	62(4)	43(15)
^{120}Te	31 (6)	31(10)				62(21)
^{122}Te	36.9(3)	41(14)		100(30)		81(27)

B(E2) transition strengths consistent with EFT expectations within the theoretical uncertainties (68% DOB intervals).

Electromagnetic transitions in deformed nuclei

"Complete" spectrum of ^{168}Er [Davidson *et al.*, J. Phys. G 7, 455 (1981)]



Separation of scale: $\xi << \Omega << \Lambda$

Key features of geometric collective model

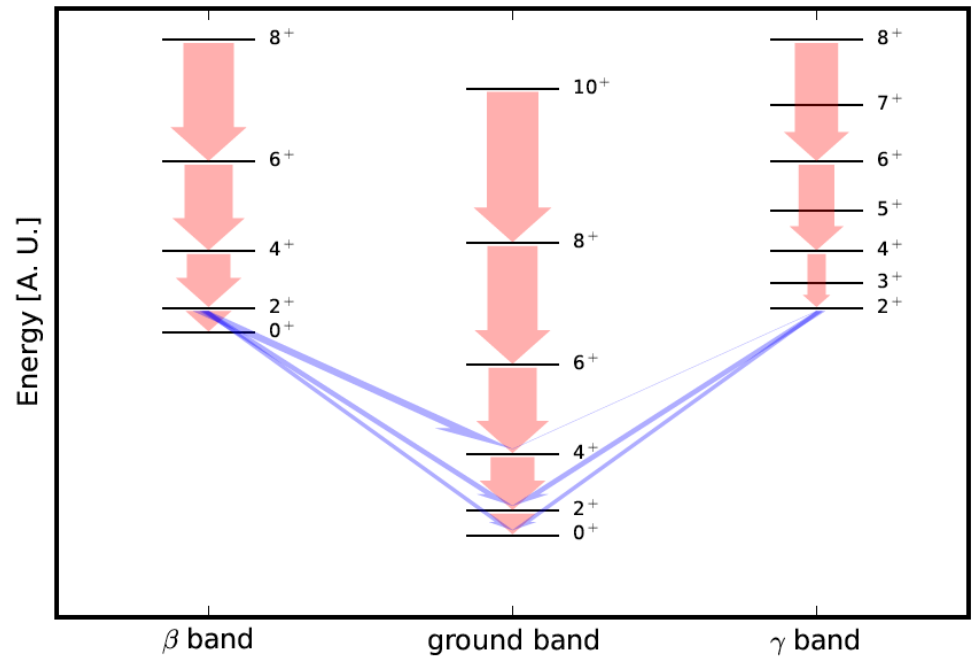
✓ Rotational bands on top of vibrational band heads
✓ Strong in-band E2 transitions
✓ Weaker inter-band E2 transitions
✓ Spectra are reproduced rather well

✗ Inter-band transitions are factors 2-10 too strong [Garrett, J. Phys. G 27 (2001) R1; Rowe & Wood "Fundamentals of Nuclear Models" (2010)]

Consistent coupling of EM fields addresses this problem

Spectra and transitions in deformed nuclei

Problem: Traditional collective models overpredict faint inter-band transitions by factors 2—10.



EFT for deformed nuclei:

- Separation of scale between rotations, vibrations, and fermionic effects
- Emergent breaking of rotational symmetry SO(3)→ SO(2) requires rotational invariance to be realized nonlinearly. [TP 2011; TP & Weidenmüller 2014/2015]
- Quadrupole degrees of freedom

EFT for deformed nuclei

Spectrum of ground-state band

$$E(I) = \frac{I(I+1)}{2C_0} - \frac{C_2}{4C_0^4}[I(I+1)]^2$$

Strength of quadrupole transitions $I_i \rightarrow I_i - 2$ in ground-state band
(Clebsch-Gordan coefficient divided out)

$$Q_{if}^2 = Q_0^2\left[1 + \frac{b}{a} I_i(I_i - 1)\right]$$

No surprises here: the EFT reproduces well known results from phenomenological models (e.g. Variable Moment of Inertia, Mikhailov theory…)

EFT provides us with insight in scale of parameters in expansion of observables

E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)

EFT: expansion parameter & naturalness

Natural sizes as expected!

	System	Expansion parameter: E_{rot} / E_{vib} $(\xi/\omega)^2$	Natural LECs: spectrum C_2/C_0^3	Natural LECs: transitions b/a
Molecules	N_2	0.000 026	0.000 006	-0.000 011
	H_2	0.0062	0.0015	0.0022
Rotational nuclei	^{236}U	0.0043	0.0011	—
	^{174}Yb	0.0026	0.0010	—
	^{168}Er	0.0094	0.0010	—
	^{166}Er	0.011	0.0020	—
	^{162}Dy	0.0083	0.0017	—
	^{154}Sm	0.0056	0.0033	—
Transitional nuclei	^{188}Os	0.06	0.012	0.008
	^{154}Gd	0.033	0.013	0.006
	^{152}Sm	0.032	0.013	0.003
	^{150}Nd	0.037	0.017	0.011

less rigid rotor

E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)