Nuclear structure and the "proton radius puzzle"

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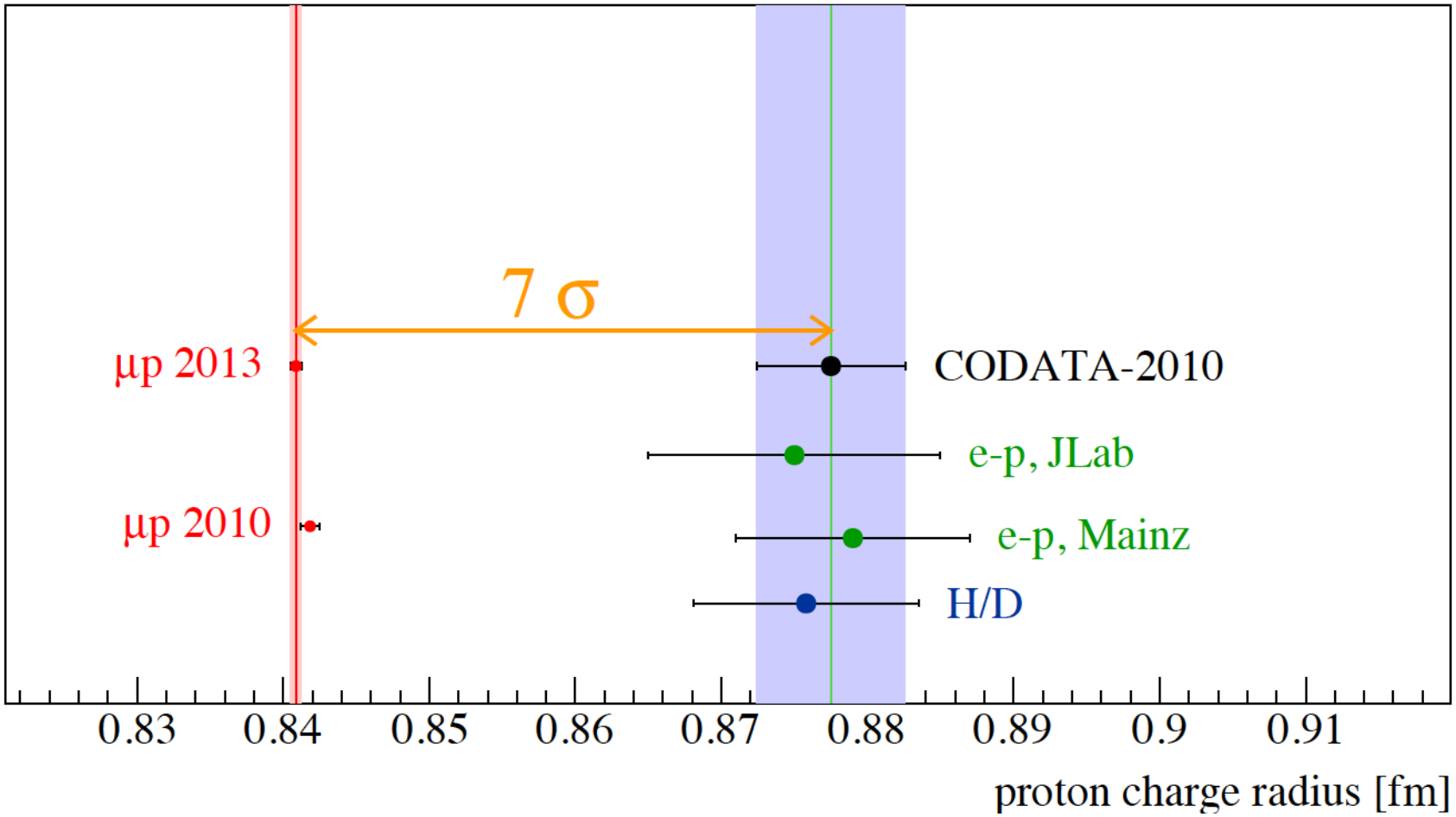
NS2016 — Knoxville

How big is the proton?



R. Pohl & J. Krauth @ CREMA

Ongoing Experiments

μ-spectroscopy experiments

- **CREMA @ PSI**
  Extract precise charge radii from Lamb shift in:
  - μD (measured, submitted: **similar discrepancy! new puzzle?)**
  - $\mu\,^4$He$^+$ (measured, finalizing: **agreement !?)**
  - $\mu\,^3$He$^+$ (measured, analyzing: $\rightarrow$ **???)**
  - $\mu\,^3$H, $\mu\,^6$He$^+$, $\mu\,^{6,7}$Li^{+2} ... (possible?)
    $\Longrightarrow$ proton puzzle, QED tests, He isotope shift, nuclear *ab initio*, ...

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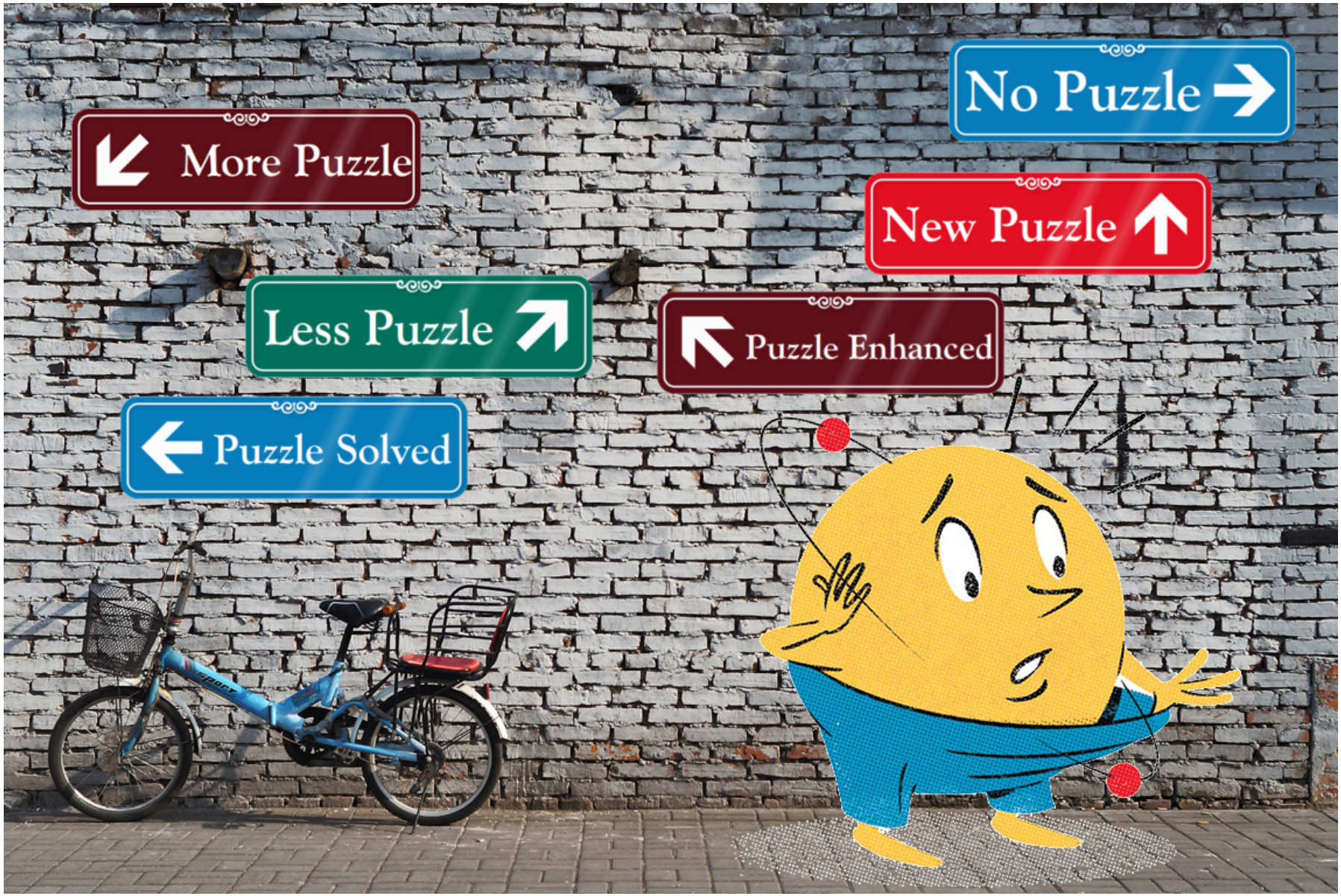
  Extract magnetic radii from Hyperfine splitting (HFS) in:
  - μH & $\mu\,^3$He$^+$ (approved)

- **@ RIKEN-RAL / (J-PARC ?)**
  - HFS in μH (planned)

Require accurate theoretical inputs from QED, hadron and nuclear physics

The proton radius puzzle



Lamb shift, charge radius & nuclear TPE

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\left(R_c\right) + \delta_{\mathrm{TPE}}$$

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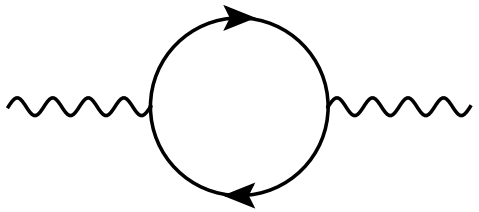
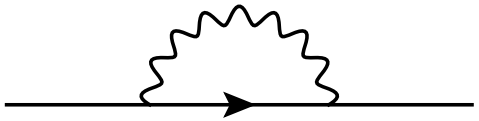
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects





- Theory of μ-p, D, ^{3,4}He$^+$ reexamined

  Martynenko *et al.* **'07**, Borie **'12**, Krutov *et al.* **'15**

  Karshenboim *et al.* **'15**, Krauth *et al.* **'15** ...

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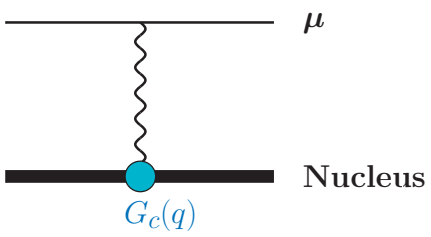
$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\left(R_c\right) + \delta_{\mathrm{TPE}}$$

- Nuclear structure corrections
  (One-Photon Exchange)
  - finite-size effect: $\delta_{size} \propto R_c^2$



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Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

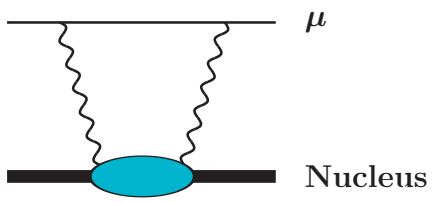
$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}(R_c) + \delta_{\mathrm{TPE}}$$

- Nuclear structure corrections
  (Two-Photon Exchange)
  - $\delta_{\mathrm{TPE}} = ?$

    $\Rightarrow$ related to nuclear response functions:

    $$S_O(\omega) = \sum\!\!\!\!\!\!\int |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

    $\Rightarrow$ can be extracted from data (very imprecise)
    $\Rightarrow$ or calculated (continuum few-body problem)



Lamb shift, charge radius & nuclear TPE

The accuracy of R_c is limited by $\delta_{\rm TPE}$

Example — μD:

$$\begin{aligned}
\Delta E^{\rm LS}_{\rm QED} &= 228.77356(75)\,{\rm meV} \\
\Delta E^{\rm LS}_{\rm rad.-dep.} &= -6.11025(28)\,r_{\rm d}^2\,{\rm meV/fm^2} + 0.00300(60)\,{\rm meV} \\
\Delta E^{\rm LS}_{\rm TPE} &= 1.70910(2000)\,{\rm meV}
\end{aligned}$$

J. Krauth *et al.* (CREMA), **Ann. Phys. (Mar. 2016)**

Status — $\mu^{3,4}$He$^+$:

- Uncertainty in $\delta_{\rm TPE}$: $\sim 20\%$
- Required: $\sim 5\%$
  (to determine R_c with $\sim 10^{-4}$ accuracy)

Calculation Methods

We have performed the first *ab-initio* calculation of δ_{TPE}

we use state-of-the-art nuclear forces

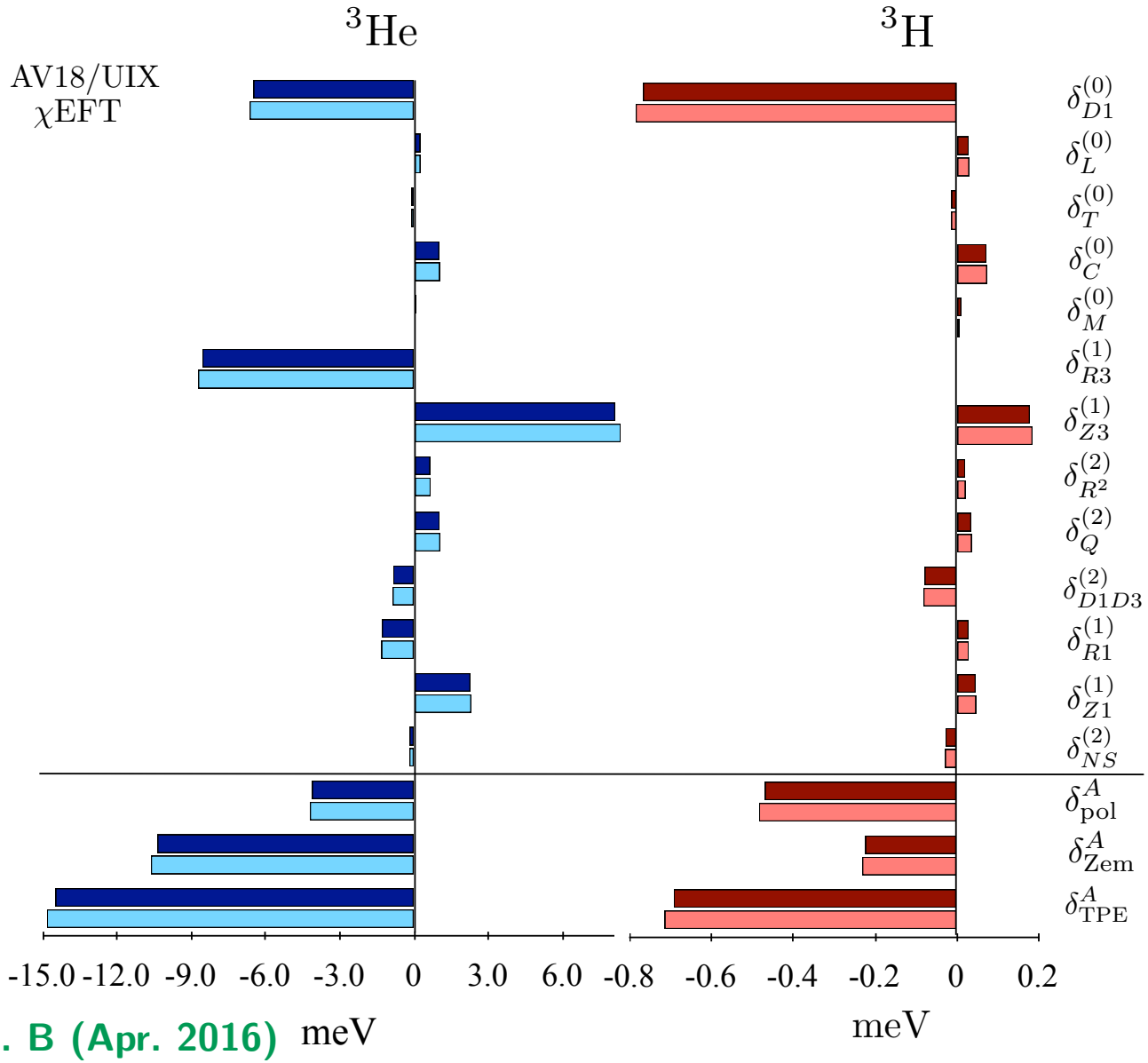
- **AV18+UIX**
- **χEFT**

$\Longrightarrow$ estimate nuclear physics uncertainty

we employ established few-body methods

- **EIHH** (bound method)
- **LIT** (continuum method)
- **LSR** (A new method)
  NND *et al.*, **Phys. Rev. C (2014)**

Results: $A = 3$



NND *et al.*, **Phys. Lett. B (Apr. 2016)**

Summary of results: $2 \leqslant A \leqslant 4$

System	Our Ref.	Unc.	Experimental Status
$\mu\,^2\mathrm{H}$	Phys. Lett. B '14	1% → 1.3%	measured, submitted
$\mu\,^4\mathrm{He}^+$	Phys. Rev. Lett. '13	20% → 6%	measured, unpublished
$\mu\,^3\mathrm{He}^+$	Phys. Lett. B '16	20% → 4%	measured, unpublished
$\mu\,^3\mathrm{H}$	Phys. Lett. B '16	4%	measurable?

- Our results agree with other values and are more accurate

⇒ Unc. comparable with $\sim 5\%$ experimental needs
⇒ Will improve precision of R_c from Lamb shifts
⇒ May help shed light on the "proton radius puzzle"

Work in progress

The work is not completed yet ...

Outlook

- Study higher-order terms (in progress)
- Quantify & reduce nuclear physics uncertainty (in progress)
- Improve treatment of nucleon finite sizes (in progress)
- Investigate nuclear corrections in $\mu\,^6\mathrm{Li}^{+2}$, $\mu\,^6\mathrm{He}^{+}$, $\cdots$
- Investigate nuclear corrections in HFS of $\mu\,^3\mathrm{He}^{+}$